



Open Your Future
Commerce Mentorship Program

Review Session

By Rahul Agarwal

MATH 104/184

Midterm Review Session-

Notes Package

- 1) Chain Rule (3.7)
- 2) Implicit Differentiation (3.8)
- 3) Logarithmic and Exponential differentiation (3.9)
- 4) Continuous Compound Interest
- 5) Related rates (3.11)
- 6) EVT, First Derivative Test, locating local/global max/min, concavity, asymptotes (4.1-4.3)
- 7) Graph Sketching (4.3)
- 8) Applications to Demand Function

Good Formulas/Identities to keep in mind:

Trig Basic Derivatives

$$d/dx (\sin x) = \cos x$$

$$d/dx (\cos x) = -\sin x$$

Continuous Compound Growth

$$A(t) = P e^{kt}$$

Basic Differentiation Rules

$$d/dx (x^r) = r x^{r-1} \quad r \neq 0$$

$$d/dx (e^{kx}) = k e^{kx}$$

$$d/dx (C) = 0 \quad (\text{where } C \text{ is constant})$$

$$d/dx (\ln(x)) = 1/x$$

$$d/dx (\log(\text{base } b) x) = 1/\ln(b) * x$$

$$d/dx (b^x) = b^x * \ln(b)$$

Product Rule

$$d/dx (f(x) * g(x)) = f'(x) * g(x) + f(x) * g'(x)$$

Quotient Rule

$$d/dx (f(x)/g(x)) = (f'(x)g(x) - f(x)g'(x)) / (g(x))^2$$

Chain Rule

$$d/dx (f(g(x))) = f'(g(x)) * (g'(x))$$

Section 1: Chain Rule (3.7)

The Chain rule is a formula used to compute the derivative of compositions of 2 or more functions.

Theorem (version 1)

Let f, g be differentiable,

Then

$$\frac{d}{dx} [f(g(x))] = f'(g(x)) * g'(x)$$

OR

Theorem (version 2)

Let $y = f(u); u = g(x)$

Then

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

Practice

Find the derivatives of the following:

1) $f(x) = \sin(3x^2 + 7x)$

2) $g(x) = \sqrt{\sqrt{x+1} + 1}$

3) $h(x) = (\cos(x^3))^2$

Section 2: Implicit Differentiation (3.8)

We use implicit differentiation when we can't solve for terms of just y .

Practice

Find the implicit derivatives of the following:

1) $xy^3 + e^{(xy)} = 5y$

2) $(x^4 + y^4)^2 = 4yx^2$ (Bonus find the tangent line at (1,1))

3) $x^4 - 5xy + y^4 = 7x$

Section 3: Logarithmic and Exponential differentiation (3.9)

4 Law's

$$\frac{d}{dx} (e^{kx}) = ke^{kx}$$

$$\frac{d}{dx} (\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx} (\log (\text{base } b) x) = \frac{1}{\ln(b)x}$$

$$\frac{d}{dx} (b^x) = b^x * \ln(b)$$

Practice

Find the derivative of the following:

1) $f(x) = ((x^3+1)^{1/4})(\cos(x))/((x^{0.5})(\sin(x)))$

2) $h(x) = (\cos(x) + x^2)^{(e^x+x+1)}$

3) $f(x) = \ln(x^2 \ln(x))$

Section 5: Related rates (3.11)

Procedure for related rates:

- 1) Read carefully, and make a sketch if need be.
- 2) Write equation(s) relating the variables
- 3) Differentiate equations implicitly, then substitute all known variables
- 4) Solve for unknown rate.

Practice

1) Pythagoras is standing 50 km north of Plato at 4pm. Pythagoras starts walking south at 20km/h and Plato starts running west at 45km/h. At what speed are Pythagoras and Plato moving away from each other at 8pm?

2) Water is flowing into a cylindrical tank at $8\text{m}^3/\text{min}$ with height 6 metres and radius 8 metres. At what rate is the water level rising when the water level is at 4 metres?

Section 6: EVT, First Derivative Test, locating local/global max/min, concavity, asymptotes (4.1-4.3)

Extreme Value theorem

If f is continuous on a closed interval I , then f has an absolute max & min on I .

First Derivative Test

Let c be a critical point of function f .

If f' changes from positive to negative at c , then f has a local max at $x=c$.

If f' changes from negative to positive at c , then f has a local min at $x=c$.

If f' doesn't change signs at $x=c$, then f does not have a local min/max at $x=c$.

Concavity

f is concave up on I , if f' is increasing on I OR $f'' > 0$ on I .

f is concave down on I , if f' is decreasing on I or $f'' < 0$ on I .

Asymptotes (given $F(x) = P(x)/Q(x)$)

Vertical Asymptotes exist where $Q(x) = 0$

Horizontal Asymptotes exist where $\lim_{x \rightarrow +/\infty} (F(x)) = C$

Slant asymptotes exist where degree of $P(x)$ is exactly 1 more than $Q(x)$

*Divide $P(x)$ by $Q(x)$ to find the slant asymptote line.

Horizontal and Slant asymptotes are mutually exclusive, you can have one, but you can't have both!

Section 6: EVT, First Derivative Test, locating local/global maxs/mins, concavity, asymptotes (4.1-4.3) Cont.

Practice

1) Find the local (and absolute) max/mins for the function $f(x) = 3x^4 - 4x^3$ on the interval $[-5, 7]$

2) Where is $f(x) = x^4 e^{-x}$ concave up?

3) Find all asymptotes of the function $g(x) = \frac{x^3 + 4x^2 + 3x + 1}{x^2 + 6x + 5}$

Section 7: Graph Sketching (4.3)

Procedure for Graph Sketching

- 1) Find $f'(x)$ and determine all max/mins and intervals of incr/decr as well as critical points.
- 2) Find $f''(x)$ and determine all intervals of concave up/down as well as inflection points.
- 3) Find x and y intercepts.
- 4) Find all asymptotes.
- 5) Graph the function using all information.

****Remember to label all points of interest and the x-axis and y-axis****

Practice

- 1) Graph $f(x) = \frac{4x^2}{(9x^2-16)}$

Section 8: Applications to the Demand Function

Definition

$\epsilon(p) = \left(\frac{p}{q(p)}\right) * q'(p) = \%$ change in demand/ $\%$ change in price

$\epsilon(p)$ is typically a negative value because price and demand are negatively correlated.

Practice

1) If the demand function is defined by $p^2 + 2q^2 = 1000$, and the current price is \$20 and management decides to drop price by 5%, by what percentage does demand change?

a) Does Total Revenue increase when we drop the price by 5%?

2) Given that the demand equation is defined by $2p^2 + 4q^2 = 10000$, and Demand is currently at 20 units growing at a steady rate of 5 units per year. At what rate is price changing (what does the sign mean?)