MATH 104/184

Midterm Review Session- Notes Package (Created by Rahul Agarwal)

1) Chain Rule (3.7)

2) Implicit Differentiation (3.8)

3) Logarithmic and Exponential differentiation (3.9)

4) Continuous Compound Interest

5) Related rates (3.11)

6) EVT, First Derivative Test, locating local/global max/min, concavity, asymptotes (4.1-4.3)

7) Graph Sketching (4.3)

8) Applications to Demand Function

Good Formulas/Identities to keep in mind:

Trig Basic Derivatives
d/dx (sinx) = cosx
d/dx (cosx) = -sinx

Continous Compound Growth
A(t) =Poe^(kt)

Basic Differentiation Rules
d/dx (x^r) = rx^(r-1) r ≠ 0
d/dx (e^kx) = ke^kx
d/dx (C) = 0 (where C is constant)
d/dx (In(x)) = 1/x
d/dx (log (base b) x) = 1/In(b)*x
d/dx (b^x) = b^x * In(b)

Product Rule
d/dx (f(x)*g(x)) = f'(x)*g(x) + f(x)*g'(x)

Quotient Rule
d/dx (f(x)/g(x)) = f'(x)g(x)-f(x)g'(x)/ (g(x))^2

Chain Rule
d/dx (f(g(x)) = f'(g(x))*(g'(x))

Section 1: Chain Rule (3.7)

The Chain rule is a formula used to compute the derivative of compositions of 2 or more functions.

Theorem (version 1)	OR	Theorem (version 2)
Let f,g be differentiable, Then d/dx [f(g(x)) = f'(g(x))*g'(x)		Let y = f(u); u=g(x) Then Dy/dx = dy/du * du/dx

Practice

Find the derivatives of the following:

1) f(x) = sin (3x^2 +7x)

2) g(x) = sqrt (sqrt(x+1) +1)

3) h(x) = (cos(x^3))^2

Section 2: Implicit Differentiation (3.8)

We use implicit differentiation when we <u>can't</u> solve for terms of just y.

Practice

Find the implicit derivatives of the following:

1) $xy^3 + e^{(xy)} = 5y$

2) $(x^4 + y^4)^2=4yx^2$ (Bonus find the tangent line at (1,1))

3) $x^4 -5xy+y^4=7x$

Section 3: Logarithmic and Exponential differentiation (3.9)

4 Law's
d/dx (e^kx) = ke^kx
d/dx (In(x)) = 1/x
d/dx (log (base b) x) = 1/In(b)x
d/dx (b^x) = b^x * In(b)

Practice

Find the derivative of the following:

1) f(x) = ((x^3+1)^(1/4))(cos(x))/((x^0.5)(sin(x)))

2) h(x) = (cos(x) +x^2)^(e^x+x+1)

3) f(x) = In(x^2In(x))

Section 4: Continuous Compound Interest

Theorem
$A(t) = Pe^{(rt)}$
P is principle amount at t= 0
r is the interest rate
t is the time
A(t) = amount after t years of continuous compound interest.

1) Froggen has invested $9000 from his winnings at the Season 4 World Championships in a mutual fund offered by Kabum Investments. Froggen has found out that his investment is growing at an astounding continuous compounded rate of 18%. Confused, he has come to you wondering what his investment would be worth at time t.

   a. Build a model that reflects the value of his investment at time t.

   b. Froggen wants to know how many years will his original investment to be doubled? Tripled?

   c. Froggen is offered a new mutual fund (of the same risk) by now billionaire, Mr. Nashor, that promises to play $16,600 in 3 years if he invests the same $9000, compounded continuously. Is he getting a better offer than the one made by Kabum investments? [calculator needed]

Section 5: Related rates (3.11)

Procedure for related rates:

1) Read carefully, and make a sketch if need be.
2) Write equation(s) relating the variables
3) Differentiate equations implicitly, then substitute all known variables
4) Solve for unknown rate.

Practice

1) Pythagoras is standing 50 km north of Plato at 4pm. Pythagoras wants to tell Plato a secret formula but Plato isn't interested in hearing it. So Pythagoras starts walking south at 20km/h and Plato starts running west at 45km/h. At what speed are Pythagoras and Plato moving away from each other at 8pm? (HINT: What formula is Pythagoras famous for?)

2) Water is flowing into a cylindrical tank at 8m^3/min with height 6 metres and radius 8 metres. At what rate is the water level rising when the water level is at 4 metres?

Section 6: EVT, First Derivative Test, locating local/global max/min, concavity, asymptotes (4.1-4.3)

Extreme Value theorem

If f is continuous on a closed interval I, then f has an absolute max & min on I.

First Derivative Test

Let c be a critical point of fuction f.
If f' changes from positive to negative at c, then f has a local max at x=c.
If f' changes from negative to positive at c, then f has a local min at x=c.
If f' doesn't change signs at x=c, then f does not have a local min/max at x=c.

Concavity

F is concave up on I, if f' is increasing on I OR f''>0 on I.
F is concave down on I, if f' is decreasing on I or f''<0 on I.

Asymptotes (given F(x)=P(x)/Q(x))

Vertical Asymptotes exist where Q(x)=0
Horizontal Asymtotes exist where lim (x->+/-inf)(F(x))=C
Slant asymptotes exist where degree of P(x) is exactly 1 more than Q(x)
*Divide P(x) by Q(x) to find the slant asymptote line.
Horizontal and Slant asymptotes are mutually exclusive, you can have one, but you can't have both!

Section 6: EVT, First Derivative Test, locating local/global max/min, concavity, asymptotes (4.1-4.3) Cont.

Practice

1) Find the local (and absolute) max/mins for the function $f(x) = 3x^4-4x^3$ on the interval [-5,7]

2) Where is $f(x) = x^4e^{-x}$ concave up?

3) Find all asymptotes of the function $g(x)= (x^3+4x^2+3x+1)/(x^2+6x+5)$

Section 7: Graph Sketching (4.3)

Procedure for Graph Sketching

1) Find f'(x) and determine all max/mins and intervals of incr/decr as well as critical points.
2) Find f''(x) and determine all intervals of concave up/down as well as inflection points.
3) Find x and y intercepts.
4) Find all asymptotes.
5) Graph the function using all information.

****Remember to label all points of interest and the x-axis and y-axis*****

Practice

1) Graph $f(x)= 4x^2/(9x^2-16)$ (Given $f''(x) = (3456x^2+2048)/(9x^2-16)^3$)

Section 8: Applications to the Demand Function

Definition

$\epsilon(p) = (p/(q(p)))*q'(p)$ = % change in demand/% change in price

$\epsilon(p)$ is typically a negative value because price and demand are negatively correlated.

Practice

1) If the demand function is defined by $p^2 + 2q^2=1000$, and the current price is $20 and management decides to drop price by 5%, by what percentage does demand change?

a) Does Total Revenue increase when we drop the price by 5%?

2) Given that the demand equation is defined by $2p^2+4q^2=10000$, and Demand is currently at 20 units growing at a steady rate of 5 units per year. At what rate is price changing (what does the sign mean?)