Rahul Agarwal
#30680128.
CMP Review session-2014.
Math 104/184.

Answer Key

Section 1. Chain Rule Practice.

1) $\frac{d}{dx} f(x) = \sin(3x^2+7x)$

1st function is $\sin(\ )$
2nd function is $3x^2+7x$.

$\Leftrightarrow f'(x) = \cos(3x^2+7x) \times (6x+7)$.

2) $\frac{d}{dx} g(x) = ((x+1)^{1/2}+1)^{1/2}$.

1st function is $(\ )^{1/2}$.
2nd function is $(\ )^{1/2}+1$.
3rd function is $x+1$.

$\Leftrightarrow g'(x) = \frac{1}{2}((x+1)^{1/2}+1)^{-1/2} \cdot (\frac{1}{2}(x+1)^{-1/2}+0) \cdot (1+0)$.

$\Leftrightarrow g'(x) = \frac{1}{4}((x+1)^{1/2}+1)^{-1/2}((x+1)^{-1/2})$

3) $\frac{d}{dx} h(x) = (\cos(x^3))^2$.

1st function $(\ )^2$.
2nd function $\cos(\ )$.
3rd function x^3.

$\Leftrightarrow h'(x) = 2(\cos(x^3))(-\sin(x^3))(3x^2)$.

$\Leftrightarrow h'(x) = -6x^2(\cos(x^3))(\sin(x^3))$.

Section 2 Implicit Differentiation

1) $\frac{d}{dx} xy^3 + \frac{d}{dx} e^{xy} = \frac{d}{dx} 5y$.

$\Leftrightarrow y^3 + 3xy^2 \cdot y' + ye^{xy} + xy'e^{xy} = 5y'$

$\Leftrightarrow 3xy^2y' + xy'e^{xy} - 5y' = -(y^3+ye^{xy})$.

$y'(3xy^2 + xe^{xy} - 5) = -(y^3+ye^{xy})$

$$y' = \frac{-(y^3+ye^{xy})}{(3xy^2+xe^{xy}-5)}.$$

Rahul Agarwal
#30680278
CMP Review Session - 2014
Math 104/184.

Answer Key

2). $\frac{d}{dx}(x^4 + y^4)^2 = \frac{d}{dx} 4yx^2.$

$\Leftrightarrow 2(x^4 + y^4)(4x^3 + 4y^3y') = 4x^2y' + 8xy.$

... Just leave it ..... OR if you really want to solve

⁝ (many hours later).

$$\Leftrightarrow y' = \frac{-2x(y^4x^2 - y + x^6)}{2y^7 + 2y^3x^4 - x^2}.$$

Bonus part.

We are given point (1, 1).

now find y' or 'm' at (1, 1).

$$\Leftrightarrow y' = \frac{-2(1 - 1 + 1)}{2 + 2 - 1} = \frac{-2}{3}$$

(notice you can plug into $2(x^4+y^4)(4x^3+4y^3y') = 4x^2y'+8xy$ and obtain same result FASTER!).

$$\Leftrightarrow \underbrace{y - 1 = \frac{-2}{3}(x-1)}_{\text{easier and time-efficient.}} \text{ or } y = \frac{-2x+5}{3}.$$

3) $\frac{d}{dx} x^4 + \frac{d}{dx}(-5xy) + \frac{d}{dx} y^4 = \frac{d}{dx} 7x.$

$\Leftrightarrow 4x^3 + (-5y + -5xy') + 4y^3y' = 7.$

$\Leftrightarrow 4x^3 - 5y - 5xy' + 4y^3y' = 7.$

$\Leftrightarrow -5xy' + 4y^3y' = 7 - 4x^3 + 5y.$

$\Leftrightarrow y'(-5x + 4y^3) = 7 - 4x^3 + 5y.$

$$\Leftrightarrow y' = \frac{7 - 4x^3 + 5y}{-5x + 4y^3}.$$

Rahul Agarwal
#30680128
CMP Review Session - 2014.
Math 104/184.

Answer Key.

Section 3: logarithmic and exponential differentiation.

1) $f(x) = (x^3+1)^{1/4}(\cos x) \,/\, (x^{1/2})(\sin x)$

$$\Leftrightarrow \frac{d}{dx}\left(\ln(f(x)) = \tfrac{1}{3}\ln(x^3+1) + \ln(\cos x) - \tfrac{1}{2}\ln(x) - \ln(\sin x)\right)$$

$$\Leftrightarrow \frac{f'(x)}{f(x)} = \frac{1}{3}\left(\frac{3x^2}{x^3+1}\right) + \frac{-\sin x}{\cos x} - \frac{1}{2}\left(\frac{1}{x}\right) - \frac{\cos x}{\sin x}.$$

$$\Leftrightarrow \frac{f'(x)}{f(x)} = \frac{3x^2}{3(x^3+1)} - \frac{\sin x}{\cos x} - \frac{1}{2x} - \frac{\cos x}{\sin x}.$$

$$\Leftrightarrow f'(x) = \frac{\left((x^3+1)^{1/4}(\cos x)\right)}{(x^{1/2})(\sin x)}\left(\frac{3x^2}{3(x^3+1)} - \frac{\sin x}{\cos x} - \frac{1}{2x} - \frac{\cos x}{\sin x}\right)$$

2). $h(x) = (\cos x + x^2)^{e^x + x + 1}.$

$$\Leftrightarrow \frac{d}{dx}\left(\ln(h(x)) = (e^x + x + 1)\ln(\cos x + x^2)\right)$$

$$\Leftrightarrow \frac{h'(x)}{h(x)} = (e^x+1)\ln(\cos x + x^2) + \frac{(e^x+x+1)(-\sin x + 2x)}{\cos x + x^2}$$

$$\Leftrightarrow h'(x) = (\cos x + x^2)^{(e^x + x + 1)} \cdot \left[(e^x+1)\ln(\cos x + x^2) + \frac{(e^x+x+1)(2x - \sin x)}{\cos x + x^2}.\right]$$

3). $f(x) = \ln(x^2\ln(x))$

$$\Leftrightarrow f'(x) = \frac{1}{x^2\ln(x)} \cdot \left(2x\ln(x) + \frac{x^{\cancel{2}}}{\cancel{x}}\right)$$

$$\Leftrightarrow f'(x) = \frac{1}{x^2\ln(x)} \cdot (2x\ln x + x)$$

$$\Leftrightarrow f'(x) = \frac{2\ln x + 1}{x\ln x}$$

Rohel Agarwal
#30680128
CMP Review Session-2014
Math 104/184.

Section 4: Continuous Compound Interest.

1.a) $A(t) = 9000e^{0.18t}$

b) i) doubled.

$\Leftrightarrow 18{,}000 = 9000e^{0.18t}$

$\Leftrightarrow 2 = e^{0.18t}$

$\Leftrightarrow \ln 2 = 0.18t$

$\Leftrightarrow \frac{\ln 2}{0.18}$ yrs $= t$

$\Leftrightarrow$ 3.85 years ~ 4 years.

ii) tripled.

$\Leftrightarrow 27000 = 9000e^{0.18t}$

$\Leftrightarrow 3 = e^{0.18t}$

$\Leftrightarrow \ln 3 = 0.18t$

$\Leftrightarrow \frac{\ln 3}{0.18}$ yrs $= t$

$\Leftrightarrow$ 6.10 years or ~ 7 years.

Calculator ready answer is fine.

c). $\Leftrightarrow 16{,}600 = 9000e^{3r}$

$\Leftrightarrow \frac{16{,}600}{9000} = e^{3r}$

$\Leftrightarrow r = \frac{\ln\left(\frac{16{,}600}{9{,}000}\right)}{3} = 0.204 = 20.4\%$.

$\therefore$ Since 20.4% > 18%, Yes it's a better offer than the one made by Kabum investments.