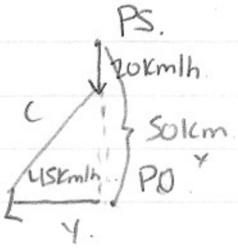
Rohel Agarwal
#30680128
CMP Review Session-2014.
Math 104/184

Answer Key

Section 5: Related Rates.

1).



W N E S

4pm → 8pm
4 hours gap.

Use pythagorus's formula.

$\Leftrightarrow C = \sqrt{x^2 + y^2}$

$\Leftrightarrow c^2 = x^2 + y^2.$

$\Leftrightarrow c^2 = y^2 + (50 - x)^2$

*C, y, and x are function of time.

$\Leftrightarrow \frac{d}{dt}\left((c(t))^2 = (y(t))^2 + (50 - x(t))^2 \right.$

* need to find $c'(t)$ *

$\Leftrightarrow \cancel{2}c(t)c'(t) = \cancel{2}y(t)y'(t) + \cancel{2}(50 - x(t))(-x'(t))$

$\Leftrightarrow c(t)c'(t) = y(t)y'(t) + (50 - x(t))(-x'(t))$

$\Leftrightarrow c'(t) = \dfrac{y(t)(y'(t) + (50 - x(t))(-x'(t)))}{c(t)}$

* $y(t) = 45t$
$x(t) = 20t$
so
$y'(t) = 45.$
$x'(t) = 20.$ *

$(200-20)^2$
$40000 - 8000 + 400$
$32,400$
$(30)^2 = 900$
$33,300$

$$\boxed{\begin{aligned} y(4) &= 4 \times 45 = 180\text{km} \\ x(4) &= 4 \times 20 = 80\text{ Km} \\ C(4) &= \sqrt{180^2 + (50-80)^2} \\ C(4) &= \sqrt{180^2 + 30^2}. \\ &= \sqrt{33,300}. \end{aligned}}$$

$\Leftrightarrow C'(4) = \dfrac{(180)(45) + (50 - 80)(-20)}{\sqrt{33,300}}$

$\Leftrightarrow C'(4) = 47.68\text{km/h}.$

Rahul Agarwal
#30680028.
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2). $V = \pi r^2 h$.

$\Leftrightarrow \frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$. ← solve for this.

$\Leftrightarrow \frac{\frac{dV}{dt}}{\pi r^2} = \frac{dh}{dt}$

$\Leftrightarrow \frac{8 m^3/min}{\pi (8m)^2} = \frac{dh}{dt}$.

$\Leftrightarrow \frac{1}{\pi} m/min. = \frac{dh}{dt}$.

Section 6: EVT, First Derivative test, locating local/global max/min, Concavity, asymptotes.

1). $f(x) = 3x^4 - 4x^3$ on $[-5, 7]$.

$f'(x) = 12x^3 - 12x^2$.

$f'(x) = 12x^2(x-1)$.

$f'(x) = 0$, when $x = 0, 1$.

$+ \quad 0 \quad +$ $\quad 12x^2$

$- \quad 1 \quad +$ $\quad (x-1)$

$[-7 \quad - \quad 0 \quad - \quad 1 \quad + \quad 5]$ $\quad 12x^2(x-1)$

since $f'(x)$ changes from $-$ to $+$ at $x=1$, $f(1)$ is local min.
Since $f'(x)$ is decreasing from $x=-7$, $f(-7)$ might be an absolute max
Since $f'(x)$ is increasing to $x=5$, $f(5)$ might be absolute max.

$f(1) = -1$.

$(1,-1)$ is local AND ABSOLUTE MIN.

$f(-7) = 8575$ ✓✓ $\quad f(5) = 1375$.

$(-7, 8575)$ is absolute MAX.

Rahul Agarwal
#30680178
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2). $f(x) = x^4 e^{-x}$ * Concave up means where $f''(x) > 0$.

$f'(x) = 4x^3e^{-x} + -x^4e^{-x}$.

$\Leftrightarrow f''(x) = 12x^2e^{-x} - 4x^3e^{-x} - 4x^3e^{-x} + x^4e^{-x}$

$\Leftrightarrow f''(x) = x^2e^{-x}(12 - 4x - 4x + x^2)$.

$\Leftrightarrow f''(x) = x^2e^{-x}(x^2 - 8x + 12)$.

$\Leftrightarrow f''(x) = x^2e^{-x}(x-2)(x-6)$

$+ \;|_0\; +$: x^2e^{-x}.

$+ \;|_2\; - \;|_6\; +$: $(x-2)(x-6)$.

$+ \;|_0\; + \;|_2\; - \;|_6\; +$: $x^2e^{-x}(x-2)(x-6)$.

$f(x)$ is Concave up in $(-\infty, 0) \cup (0,2) \cup (6, \infty)$

3). $g(x) = \dfrac{x^3 + 4x^2 + 3x + 1}{x^2 + 6x + 5} = \dfrac{x^3 + 4x^2 + 3x + 1}{(x+1)(x+5)}$

vertical asymptotes (denominator = 0).

$x^2 + 6x + 5 = 0$.

$(x+5)(x+1) = 0$.

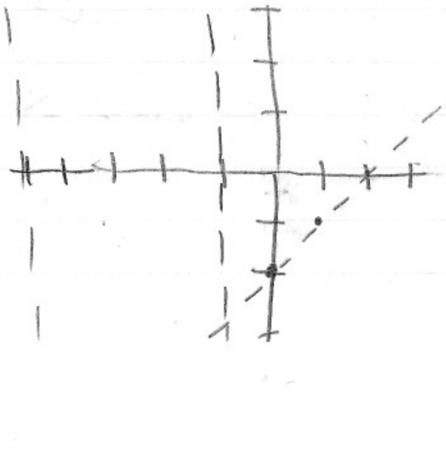
$x = -5, -1$.

horizontal asymptotes

$\lim_{x \to \pm\infty} g(x) = \dfrac{x^3}{x^2} = x = \infty$

slant asympote (since degree of numerator is 1 bigger than denominator)

$$\begin{array}{r} \boxed{x-2} \\ x^2+6x+5 \;\overline{)\; x^3 + 4x^2 + 3x + 1} \\ -(x^3 + 6x^2 + 5x) \\ \hline -2x^2 - 2x + 1 \\ -(-2x^2 - 12x - 10) \\ \hline 10x + 11. \end{array}$$



Rahul Agarwal
#30680478
CMP Review Session
Math 104/184

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3). $f(x) = \dfrac{4x^2}{9x^2-16} = \dfrac{4x^2}{(3x+4)(3x-4)}$

$\Rightarrow f'(x) = \dfrac{8x(9x^2-16) - 18x(4x^2)}{(9x^2-16)^2}$

$\Rightarrow f'(x) = \dfrac{\cancel{72x^3} - 128x - \cancel{72x^3}}{(9x^2-16)^2}$

$\Rightarrow f'(x) = \dfrac{-128x}{(9x^2-16)^2}$

$f''(x) = \dfrac{3456x^2+2048}{(9x^2-16)^3}$

$+ \mid -$ $\quad -128x$
0

$+ \; + \; + \; +$ $\quad (9x^2-16)^2$
$-4/3 \quad 4/3$

incr $\quad$ incr $\quad$ decr $\quad$ decr
$+ \quad + \quad - \quad -$ $\quad f'(x)$
$-4/3 \quad 0 \quad 4/3$
max.

$18x$

$+ + + +$ $\quad 3456x^2+2048$

CU $\quad$ CD $\quad$ CU.
$+ \quad - \quad +$ $\quad (9x^2-16)^3$
$-4/3 \quad 4/3$

asymptotes.

vertical asymptote (den = 0.

$9x^2-16=0$

$x = \pm 4/3$.

horizontal asymptote

$\lim_{x\to\pm\infty} f(x) = \dfrac{4\cancel{x^2}}{9\cancel{x^2}} = 4/9$

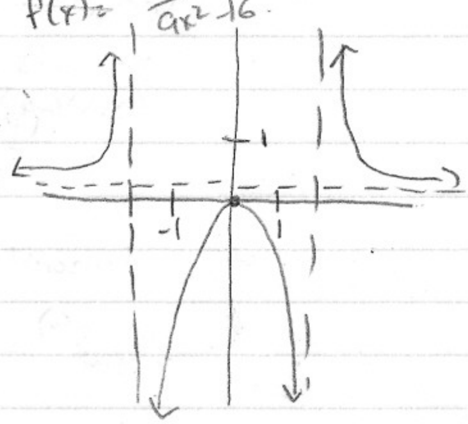
x-inter $\Rightarrow f(x)=0$

$0 = \dfrac{4x^2}{9x^2-16} \rightarrow x=0$.

y-inter $\rightarrow x=0$.

$\dfrac{4(0)^2}{9(0)^2-16} \Rightarrow y=0$.

$f(x) = \dfrac{4x^2}{9x^2-16}$.



Rahul Agarwal
#30680228
CMP Review Session
Math 1041184.

Answer Key

Section 8 : Applications to the Demand function

1). $p^2 + 2q^2 = 1000$

$20^2 + 2q^2 = 1000$

$q = \sqrt{300}$ or $10\sqrt{3}$

$\left(p^2 + 2q^2 = 1000\right)\frac{d}{dp}$

$2p + 4qq' = 0.$

$q' = \frac{-2p}{4q} = \frac{-p}{2q}$

So $\mathcal{E}(p) = \frac{p}{q} \times \frac{-p}{2q}$

$\mathcal{E}(p) = \frac{\cancel{20}}{10\sqrt{3}} \cdot \frac{-\cancel{20}}{2 \times 10\sqrt{3}} = -\frac{2}{3}.$

$-\frac{2}{3} = \frac{\%\Delta \text{ demand}}{\%\Delta \text{ price}} \Rightarrow -\frac{2}{3} = \frac{x}{-5\%} \Rightarrow x = \frac{1}{30}$ or $3.\overline{33}\%$ increase.

a) $|\mathcal{E}(p)| < 1$, Price inelastic, therefore revenue increase (Yes).

2). step 1 find p.

$2p^2 + 4q^2 = 10{,}000$

$2p^2 + 4(20)^2 = 10{,}000$

$p = \sqrt{4200} = 10\sqrt{42}$

$\frac{d}{dt}\left(2p^2 + 4q^2 = 10{,}000\right)$

$4pp' + 8qq' = 0.$

$p' = \frac{-8qq'}{4p}$

$p' = \frac{-2qq'}{p}$

$p' = \frac{-2(20)(5)}{10\sqrt{42}}$

$p' = \$\frac{-20}{\sqrt{42}}$ / year.

price is dropping