commerce undergraduate society

MATH104/184 Review Session

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CMP

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I. Introduction and Few Tips

Hello!
I hope you are having a great term so far.

Here are few great tips for your MATH 104/184 Midterm:

1. ***Practice, practice, and practice*.**

2. ***If you don't know how to solve a question, come back to it.***
   The midterm can be very long and the time given to you may be very short. Don't waste your time on one question.

3. ***Make sure you write down all the information that is given to you. (or highlight some important points)***
   Sometimes, when you are nervous, it is easy to make mistakes.

4. ***Watch out for signs***
   Remember, negative and positive signs can result in very different answers.

If you have any question, feel free to ask! I am more than happy to help you out. Good Luck!

II. Review of Exponentials, Logarithms, and Inverse Functions

Things to Remember:

$b^x * b^y = b^{x+y}$ $b^x / b^y = b^{x-y}$ $(b^x)^y = b^{x*y}$	$y = b^x \leftrightharpoons \log_b y = x$ $\log_b(xy) = \log_b x + \log_b y$ $\log\left(\frac{x}{y}\right) = \log x - \log y$ $\log(x^y) = y\log(x)$

Question 2(a): Find the inverse of $f(x)=x^3-9$

Question 2(b): A culture of bacteria has a population of 150 cells when it is first observed. The population doubles every 12hr. How long does it take the population to reach 10,000?

III. A Standard Business Problem

Things to Remember:

Cost Function C = C(q), where p= price and q=quantity demanded
Revenue Function R=R(q)=p*q
Break-even Points: C(q)=R(q)
Profit Function P(q)=R(q)-C(q)

Question 3: Jenny decides to start a new cookie business. She estimates that when the price of the cookie is $20, then the daily demand for it is 500 units. For every $1 increase in the price, the daily demand decreases by 5 units. Assume that the fixed costs of production on a daily basis are $10,000, and the variable costs of production are $7 per unit.

(a) Find the linear demand equation for the cookie. Use the notation p for the unit price and q for the daily demand.

(b) Find the weekly cost function, C=C(q), for producing q cookies per day.

(c) Find the daily revenue function, R=R(q).

(d) Find the break-even points where Cost equals Revenue.

(e) How should Jenny operate in order to maximize the daily profit $P=P(q)$?

IV. Limits

Question 4(a): Evaluate the limit

$$\lim_{x\to 3}(x^2-2)(x^3+6x-1)$$

Question 4(b): Evaluate the limit

$$\lim_{x\to 2}\frac{x^2-4}{x-2}$$

Question 4(c): Evaluate the limit

$$\lim_{x\to 0}\frac{(2+x)^3-8}{x}$$

V. Continuous Functions

Things to Remember:

In order for the function f(x) to be continuous at a, the following conditions must be met:

- **f(a) is defined**
- $\lim_{x \to a} f(x)$ **exists (that is, the right sided and left sided limit must equal)**
- $\lim_{x \to a} f(x) = f(a)$

Question 5(a): If f and g are continuous functions with f(0)=6 and

$\lim_{x \to 0} f(x)g(x) = 36$, find g(0).

Question 5(b): For what values of c is the function g continuous on (-∞,∞)?

$$g(x) = x^2 \text{ if } x < 2,$$
$$= cx + 20 \text{ if } x \geq 2$$

VI. Intermediate Value Theorem

Things to Remember:

Intermediate Value Theorem: f(x) is continuous on interval [a,b] and L is a number strictly between f(a) and f(b). Then there exists at least one number c in (a,b) satisfying f(c)=L. (Or, if f is continuous on [a,b], then the graph of f(x) must cross the horizontal line y=L at least once).

Question 6: Suppose you invest $100 in a 10-year savings account with a fixed annual interest rate r, with yearly compounding. The amount of money A in the account after 10 years is $A(r) = 100(1 + r)^{10}$. You want to have $1000 in the account after 10 years.

(a) Use the Intermediate Value Theorem to show there is an interest rate between 0% and 40% for which A(r)=1000.

VII. Rules of Differentiation

Things to Remember:

Derivative: Slopes of the tangent line = instantaneous rate of change in f at a
The average rate of change in f on the interval [a,a+h] is the slope of the corresponding secant line:

$$m_{sec} = \frac{[f(a+h) - f(a)]}{h}$$

The instantaneous rate of change in f at a:

$$m_{tan} = \lim_{h \to 0} \frac{[f(a+h) - f(a)]}{h}$$

Question 7(a): Find an equation of the line tangent to the graph of $\mathrm{f(x)} = \mathrm{x}^2 + 4\ at\ (1,5)$

A. Constant and Power Rules

Things to Remember:

Constant Rule: $\frac{\mathrm{d}}{\mathrm{dx}}(c) = 0$

Power Rule: $\frac{\mathrm{d}}{\mathrm{dx}}(x^n) = nx^{n-1}$

Constant Multiple Rule: $\frac{\mathrm{d}}{\mathrm{dx}}(cf(x)) = cf'(x)$

B. Sum Rule

Things to Remember:

Sum Rule: $\frac{\mathrm{d}}{\mathrm{dx}}\left(f(x) + g(x)\right) = f'(x) + g'(x)$

Question 7A&B (a): Find the values of $\frac{\mathrm{d}}{\mathrm{dz}}(9z^{10} + 6z^6 - 8z^2 + 7z + 9)$

C. Product and Quotient Rules

Things to Remember:

Product Rule: $\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{[f'(x)g(x) - f(x)g'(x)]}{g(x)^2}$

Question 7C (a): Find the values of $\frac{d}{dx} x^9(\sqrt{x-9})$

Question 7C (b): Find the values of $\frac{d}{dx}\frac{(x^2+3x+4)}{x^2-1}$

D. Chain Rule

Things to Remember:

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) * g'(x)$

Chain Rule for Powers: $\frac{d}{dx}(g(x)^n) = n(g(x))^{n-1} * g'(x)$

Question 7D (a): Find the values of $\frac{d}{dx}(6x^2 + 7x + 5)^6$

Question 7D (b): Find the values of $\frac{d}{dx}\sqrt{7x^2 + x + 1}$

VIII. Trigonometric Derivatives

Things to Remember:

$$\frac{\mathrm{d}}{\mathrm{dx}}(sinx) = \cos(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}(cosx) = -\sin(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}(tanx) = \sec^2(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}(secx) = \sec(x)\tan(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}(cotx) = -\csc^2(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{cscx}) = -\csc(x)\cot(x)$$

Question 8(a): Calculate the derivative of $y=e^{3x}\cos x$.

Question 8(b): Differentiate $y=\sin^{10}x$

Question 8(c): Differentiate $(\tan(x) + 9)^{26}$

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IX. Derivatives of e^x, Logarithms and Exponentials

Things to Remember:

$$\frac{\mathrm{d}}{\mathrm{dx}}(e^x) = e^x$$
$$\frac{\mathrm{d}}{\mathrm{dx}}\left(e^{u(x)}\right) = e^u * \frac{du}{dx}$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(e^{kx}) = ke^{kx}$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(lnx) = \frac{1}{x}$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(\ln|u(x)|) = \frac{u'(x)}{u(x)}$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(b^x) = b^x lnb$$
$$\frac{\mathrm{d}}{\mathrm{dx}}(\log_b x) = \frac{1}{xlnb}$$

Question 9(a): Find derivative of $\mathrm{y} = \frac{\mathrm{lnx}^3}{\mathrm{x}^4}$

Question 9(b): Find derivative of $y = \cos(x)\ln(cos^3 x)$

Question 9(c): Find derivative of $y = x^{x^9}$

X. Rates of Change – Velocity, Acceleration, and Marginal Cost

Question 10: The price-demand and cost functions for the production of products are the following:

$$p=210-x$$

and cost $C(x)=50000+110x$,

where x= the number of products and can be sold at a price of p dollars per unit and C(x) is the total cost ($) of producing x units.

(a) Find the marginal cost as a function of x.

(b) Find the revenue function in terms of x.

(c) Find the marginal revenue function in terms of x.

(d) Evaluate the marginal revenue function at x=100.

XI. Implicit Differentiation

Question 11(a): Find dy/dx by implicit differentiation

$$x^2 - 2xy + y^3 = 5$$

Question 11(b): Find dy/dx by implicit differentiation at (0,49)

$$\sqrt{x + y} = 7 + x^2 y^2$$

XII. Elasticity of Demand

Things to Remember:

Price Elasticity of Demand (Demand Elasticity) tells us how revenue changes as price changes.

Price Elasticity of Demand: $\frac{p}{q} * \frac{dq}{dp}$

When $|\varepsilon| > 1$, the good is price elastic ; when there is 1% change in price, there is greater than 1% change in quantity demanded

When $|\varepsilon| < 1$, the good is price inelastic; when there is 1% change in price, there is less than 1% change in quantity demanded

When $|\varepsilon| = 1$, the good is unit elastic; when there is 1% change in price, there is also 1% change in quantity demanded (but not always)

Question 12: Suppose the demand curve for cookies is given by q=400-10p

(a) Compute the price elasticity of this demand function.

(b) What is the price elasticity of demand when price is $20?

(c) What is the percentage change in the demand if the price is $20 and increases by 2%?

XIII. Exponential Growth and Compound Interest

Things to Remember:

$$\mathbf{A} = \mathbf{P}\left(\mathbf{1} + \frac{\mathbf{r}}{\mathbf{n}}\right)^{\mathbf{nt}}$$

P=principal (the amount you borrow or deposit)
r=annual rate of interest
n=number of times the interest is compounded per year
t=number of years
A= amount of money and interest accumulated after n years

Continuous Interest:

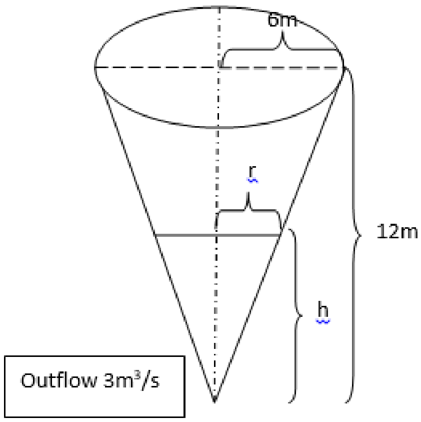
$$\mathbf{A} = \mathbf{P}(\mathbf{e})^{\mathbf{rt}}$$

Question 13(a): Today, Ara has $1000 in her bank account. If it is compounded twice a year, and there is 3% annual interest rate, how much will she have in 2 years?

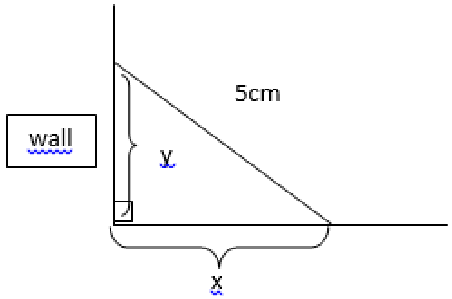
Question 13(b): Today, Ara has $1000 in her bank account. A bank is paying an annual interest rate of 3%, compounded continuously. How much will she have in 2 years?

XIV. Related Rates

Question 14(a): An inverted conical water tank with a height of 12m and a radius of 6m is drained through a hole at a rate of $3m^3/s$. What is the rate of change of the water depth when the water depth is 3m?



Question 14(b): You have a 5cm ladder learning against a wall. If the foot of the ladder slides away from the wall at 1cm/s, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 3 cm away from the wall?



XV. Maxima and Minima

Things to Remember:

Critical Points: f'(x)=0, or f'(x) does not exist but f(x) does To maximize or minimize f(x) on [a,b] evaluate f(x) at x=a, x=b, and each critical number.

Question 15: Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 18x^2 + 108x + 10$$

on the interval [0,10].