290 Review Session

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Terms and Definitions

Constraints - Restrictions or limitations imposed on a problem
(Blue Box)

Objective function - A mathematical expression that describes the problem's objective. Either Max or Min

Optimal solution - The specific decision-variable value or values that provide the "best" output for the model.
(Red Box) **Objective Cell** = Green Shade

Infeasible solution - A decision alternative or solution that does not satisfy one or more constraints.

Feasible solution - A decision alternative or solution that satisfies all constraints.

Slack variable - A variable added to the left-hand side of a less-than-or-equal-to constraint to convert the constraint into an equality. The value of this variable can usually be interpreted as the amount of unused resource.

Redundant constraint - A constraint that does not affect the feasible region. If a constraint is redundant, it can be removed from the problem without affecting the feasible region.

Extreme point - Graphically speaking, extreme points are the feasible solution points occurring at the vertices or "**corners**" of the feasible region. With two-variable problems, extreme points are determined by the intersection of the constraint lines.

Sensitivity analysis - The study of how changes in the coefficients of a linear programming problem affect the optimal solution.

Range of optimality - The range of values over which an objective function coefficient may vary without causing any change in the values of the decision variables in the optimal solution.

Range of feasibility - Provides the limits where the shadow prices are applicable.

Right-Hand-Side Allowable Increase (Decrease) - The allowable increase (decrease) of the right-hand side of a constraint is the amount the right-hand side may increase (decrease) without causing any change in the Optimal Solution. The allowable increase (decrease) for the right-hand side can be used to calculate the range of feasibility for that constraint.

Cell References

$A1 Allows the row reference to change, but not the column reference.

A$1 Allows the column reference to change, but not the row reference.

A1 Allows neither the column nor the row reference to change.

Understanding and Drawing the Objective Function and Constraints

Question 1

A student current buys supplies for his start-up company from 2 online e-commerce companies. He gets his supplies from Amazon and Ebay. Let

x = number of units of the product received from Amazon
y = number of units of the product received from Ebay

a. Write an expression for the total number of units of the product received.

b. Shipments from Amazon cost $0.20 per unit, and shipments from Ebay cost $0.25 per unit. Develop an objective function representing the total cost of shipments.

c. Assuming the monthly demand at the retail store is 5000 units, develop a constraint that requires 5000 units to be shipped.

d. No more than 4000 units can be shipped from Amazon, and no more than 3000 units can be shipped from Amazon in a month. Develop constraints to model this situation.

e. Of course, negative amounts cannot be shipped. Combine the objective function and constraints developed to state a mathematical model for satisfying the demand at minimum cost.

Bonus: Using Excel. Solve for x and y, to satisfy the demand at minimum cost.

Hone your graphical drawing skills

Drawing Constraint Lines

1) Find the solutions that satisfy the following constraints:

(Shade and indicate the feasible reason)

a. $4A + 2B <= 16$ **b.** $4A + 2B >= 16$ **c.** $4A + 2B = 16$

2) Find the optimal solution using the graphical solution procedure.

Objective Function Max 2A + 3B

s.t.

1A + 2B <= 6

5A + 3B <= 15

A, B >=0

Calculating the Optimal Solution

<u>Question 2</u>

Home Run Company, makes two different types of baseball gloves for its professionals: a regular model and a catcher's model. Like any company, it has a finite number of resources and time. Working with 4 departments, Cutting and Sewing has 900 hours of production time available, 300 hours for finishing, and 100 hours available in its packaging and shipping department. The production time requirements and the profit contribution per glove are given in the following table:

Production Time (Hours)

Model	Cutting and Sewing	Finishing	Packaging and Shipping	Profit/Glove
Regular	1	$\frac{1}{2}$	$\frac{1}{8}$	$5
Catcher's	$\frac{3}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$8

The company is only interested in maximizing the total profit contribution, answer the following:

a. What is the Objective Function for the company, subject to what constraints?

Objective Function:

s.t (Constraints)

b. Find the optimal solution using the graphical solution procedure. How many gloves of each model should *Home Run Company* manufacture?

c. What is the total profit contribution *Home Run Company* can earn with the given production quantities?

d. How many hours of production time will be scheduled in each department?

Cutting and Sewing Hours:

Finishing Hours:

Packaging and Shipping Hours:

e. Which Department is a Binding constraint? (Circle)

Cutting and Sewing Finishing Packaging and Sewing f1. Which Department is a Non-Binding constraint?

f2.What is the **Allowable Decrease** for this department? (i.e. Slack time)

f3. What is the **Allowable Increase** for this department?

f4. What is the **Shadow Price** for this department?

f5. If we decrease the number of hours for Cutting and Sewing by 176 hours, will the shadow price:

Change Remain the Same

Calculating the shadow price

<u>Question 3</u>

Lovely Potion is a small firm that operates at Hogwarts School of Wizardry. Because of its small stature, the firm is only able to survive, by specialising in two potions. In the production process, up to three raw materials are combined to produce two potions: Felix Felicitus (the lucky potion)[1] and Veritaserum (the truth potion)[2]

The potion composition, and the profit/ton, is listed in the table below.

Product	Material 1	Material 2	Material 3	Profit per ton
Felix Felicitus	$2/5$	0	$3/5$	40
Veritaserum	$1/2$	$1/5$	$3/10$	30

Lovely Potion's production is constrained by a limited availability of the three raw materials. For the current production period, Lovely Potion has the following quantities of each raw material available:

Raw material	Amount Available For Production (Tons)
Material 1	20
Material 2	5
Material 3	21

a. What is the linear programming model for this problem?

Objective Function:

s.t (Constraints)

[1] One of the most intriguing potions is Felix Felicitus, the lucky potion. This potion is described as a "pool of living gold," which constantly shifts and leaps in the cauldron it is brewed in. A small amount of the potion will render the drinker totally lucky for a set period of time: every task undertaken will be successful, and the day will be more or less perfect. The potion is forbidden to people when they are participating in sports and examinations, because it would of course cause them to perform perfectly and answer every question correctly, thus constituting a form of cheating.

[2] Veritaserum, a potion which forces the drinker to tell the truth when questioned. It is used by wizards during interrogations to ensure that they get truthful answers from the subject, and according to Snape, it is also very potent: a few drops go a long way. It is also among the more difficult potions to make, and requires the steady hand of a master.

b. Find the optimal solution using the graphical solution procedure. How many tons of each product should be produced, and what is the projected total profit contribution?

c. Is there any unused material? If so, how much?

d. Are any of the constraints **redundant**? If so, which ones?

e. How much would **Profit** increase if *Lovely Potion* could increase its access to 1 extra ton of Material 1?

f1. How much would **Profit** increase if *Lovely Potion* could increase its access to 1 extra ton of Material 2?

f2. Determine the **allowable increase** for the Material 2 constraint.

g. How much would **Profit** increase if *Lovely Potion* could increase its access to 1 extra ton of Material 3?

Understanding the Sensitivity Report

<u>Question 4</u>

Escape Da Summer Heat Company manufactures three air conditioners: an economy model, a standard model, and a deluxe model. The profits per unit are $63, $95, and $135, respectively. The production requirements per unit are as follows:

	# of Fans Motors	# of Cooling Coils	Manufacturing Time (Hours)
Economy	1	1	8
Standard	1	2	12
Deluxe	1	4	14

Winter has ended and summer looms, the company has 200 fan motors, 320 cooling coils, and 2400 hours of manufacturing time available. How many economy models (E), standard models (S), and deluxe models (D) should the company produce in order to **maximize profit**?

a. What is the Objective Function for the company, subject to what constraints?

Objective Function:

s.t (Constraints)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C15	Economy # Aircon to make	80	0	63	12	15.5
C16	Standard # Aircon to make	120	0	95	31	8
C17	Deluxe # Aircon to make	0	-24	135	24	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C9	Model Output # of Fans Motors	200	31	200	80	40
D9	Model Output # of Cooling Coils	320	32	320	80	120
E9	Model Output Manufacturing Time (Hrs)	2080	0	2400	1E+30	320

Using the Sensitivity Report Given

b. What is the **optimal solution**, and what is the value of the objective function?

c. Which constraints are **binding**?

d. Which constraint are **non-binding** (extra capacity)? How much?

e. Using only the Sensitivity Report, what happens to total profit, if the **# of fan motors** changes to:

e1) 199	Profit
e2) 201	Profit
e3) 280	Profit
e4) 281	Profit

e5) What happens to the **optimal solution** in all of the above scenarios?

Change Remain the Same

f. If the **unit-profit** for the deluxe model were increased to $150 per unit, would the **optimal solution** change?

f. Would the **optimal solution** change if the **unit-profit** for the Standard model were decreased to:

f1) $94 per unit
f2) $86 per unit

f3) Calculate the profit, under scenario f1) – when standard model profit = $94