# MATH 104/184

# MIDTERM REVIEW SESSION

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*** means a question is high difficulty level

# I. Function Review

Exponential Function:

$$b^m \times b^n = b^{m+n}$$

$$b^m \div b^n = b^{m-n}$$

$$(b^m)^n = b^{mn}$$

$$b^1 = b$$

$$b^0 = 1$$

Logarithmic Function:

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b (x^n) = n\log_b x$$

$$\log_b (b) = 1$$

$$\log_b (1) = 0$$

1. a) Solve $\log_6 x^7 = 5$ for x.

b) Your student loans of $10,000 have an annual interest rate of 12% compounded continuously. How long will it take for your debt to double?

c) Your student loans of $5,000 grew to $12,000 after 30 months. What is the effective interest rate if it was compounded annually?

## II. The "Standard Business Problem"

Remember:
p = price and q = quantity
Demand equation = p(q) slope * q + intercept
Cost Function = C(q) = Variable cost * quantity + fixed cost
Revenue Function = R(q) = p*q
Break-even Point is when R(q) = C(q)
Profit Function P(q) = R(q) – C(q)

2. A penguin decides to start a popsicle business. He estimates that when the price of a box of popsicles is $50, the monthly demand will be 300 boxes and for every $2 increase in the price, the monthly demand will fall by 10 boxes. There is also a fixed production cost of $1000 per month and the variable cost are $5 per box.

   a) Find the linear demand equation for boxes of popsicles.

   b) Find the cost function.

c) Find the monthly revenue function.

d) Find the break-even point(s), leave your answer in a calculator ready form

e) Find the optimal price to maximize profit.

f) What is his profit? Leave your answer in calculator ready form

## III. Limits

3. a) Evaluate the limit $\lim_{x \to 7} \frac{x^2-6x-7}{x^2-5x-14}$

b) Evaluate the limit $\lim_{x \to \infty} \frac{8x^9+4x^3+x^2+21}{2x^9+21x^2-4}$

c) Evaluate the limit $\lim_{x \to \infty} \frac{8x^9+4x^3+x^2+21}{2x^8+21x^2-4}$

d) Evaluate the limit $\lim_{x \to \infty} \frac{8x^8+4x^3+x^2+21}{2x^9+21x^2-4}$

c) Evaluate the limit $\lim_{x \to 3^-} \frac{|x-3|}{x-3}$

d) Let f(x) = $\begin{cases} \sin \sqrt[3]{e} & if\ x = e^{\pi} \\ e^{\pi} & if\ x \neq e^{\pi} \end{cases}$, evaluate the limit $\lim_{x \to e^{\pi}} f(f(x))$.

Give your answer in a calculator ready form

e) Evaluate the limit $\lim_{x \to 16} \frac{\sqrt{x}-4}{16-x}$

f)*** Evaluate the limit $\lim_{x \to 0} \frac{\sqrt[3]{x+1}-1}{x}$ hint use the identity (a-b)($a^2$ +ab + $b^2$) = $a^3 - b^3$

# IV. Continuous Functions

A function f(x) is continuous at point a if these three conditions are met:

1. f(a) is defined
2. $\lim_{x \to a} f(x)$ exists and the right and left sided limits are equal
3. $\lim_{x \to a} f(x)$ = f(a)

4. a) True or False:

If g(x) is not differentiable at some point a, then g(x) is not continuous at a. If false draw a graph as a counter argument.

b) For what value of a will the function f(x) continuous on all x.

$$f(x) = \begin{cases} x^3 + a & if\ x \leq \pi \\ 3a\left(\sin\frac{x}{2}\right) & if\ x > \pi \end{cases}$$

c) *** For what values of a and b will the function f(x) be continuous for all x

$$f(x) = \begin{cases} 2a + b^x & if\ x \le 0 \\ ax^3 + b(x+1) & if\ 0 < x \le 1 \\ 6a\sqrt[3]{x} & if\ x > 1 \end{cases}$$

## V. Intermediate Value Theorem

f(x) is continuous on interval [a,b] and L is a number strictly between f(a) and f(b). Then there exist at least one number c in (a,b) that satisfies f(c) = L.

5. Prove using IVT that the equation f(x) = $x^3 - 4x + 1$ has at least 1 zero.

## VI. Definition of Derivative

$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ OR

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

6. a) Using the definition of derivative to find the tangent line of f(x) = $x^2$ at x = 2

b)*** Evaluate the limit $\lim_{x \to \pi} \frac{\cos(2x) - 1}{x - \pi}$ (hint: recognize the definition of derivative of some function f(x) and derivative it)

c)****(SUPER HARD BONUS) Let k(x) = f(x)/ g(x) and using limits show that k'(x) = $\frac{[f'(x) * g(x) - g'(x) * f(x)]}{g(x)^2}$. Hint f '(x) =$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ and g'(x) =$\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

## VII. Derivatives

- **Constant Rule:** $f(x) = c$ then $f'(x) = 0$
- **Constant Multiple Rule:** $g(x) = c \cdot f(x)$ then $g'(x) = c \cdot f'(x)$
- **Power Rule:** $f(x) = x^n$ then $f'(x) = nx^{n-1}$
- **Sum and Difference Rule:** $h(x) = f(x) \pm g(x)$ then $h'(x) = f'(x) \pm g'(x)$
- **Product Rule:** $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$
- **Quotient Rule:** $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **Chain Rule:** $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$

| f(x) | f '(x) |
|---|---|
| sin x | cos x |
| cos x | - sin x |
| tan x | $\sec^2 x$ |
| $b^x$ | $b^x \ln b$ |
| $e^x$ | $e^x$ |
| $\log_b x$ | 1/(x ln b) |
| ln x | 1/x |

7. a) Find the derivative of h(x) = $x^3\sqrt{x^2+1}$. Do not simplify

b) Find the value of $(e^{x^3+x-1})'$

c) Find the derivative of cos(sin(2x+2))

d) Find the derivative of $\ln((z^3+2)^2)$

## VIII. Application of Derivatives

8. a) find the acceleration of an object whose position is $s(t) = t \sin(2t)$

b) A snowball is thrown, from a starting height of 0 meters, upwards with an initial velocity of 15m/s and its path is parabolic arc given by the function of time $h(t) = 12t - 3t^2$. What is the snowball's velocity when it hits its target at $h=0$?

## IX. Implicit Differentiation

9. a) Find dy/dx using implicit differentiation on $x^2 - 3xy + 2y^3 = 5$

b) Show that the dy/dx of $e^y$= x is equal to 1/x using implicit differentiation