commerce undergraduate society

COMM 290

MIDTERM/FINAL EXAM REVIEW SESSION

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CMP

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Vocabulary Overview

Objective function – the function describing the problem's objective which you are attempting to maximize or minimize.

Optimal solution – the best set of decisions that maximizes the objective function while remaining within the constraints.

Target Cell – Contains the output of the objective function and is highlighted in green.

Constraint – A limitation of some sort posed with the problem. Always enclosed by a blue border.

Multiple optima – There are multiple sets of optimal solutions.

Feasible region – The region in which all solutions are valid and subject to the constraints.

Infeasible solution – There is no feasible region associated with your LP.

Unbounded solution – The feasible region is infinitely large, usually due to lack of a constraint.

Input Data – The data given to you as part of a problem. Usually highlighted in yellow.

Action Plan – The "action" you will take to solve the problem, usually involving modifying decisions enclosed by a red border.

Redundant constraint – A constraint which does not contact the feasible region in any way

Non-negativity constraint – A constraint which makes sure a "decision" cannot be a negative value.

RHS Allowable Increase/Decrease of a Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint binding.

RHS Allowable Increase/Decrease of a Non-Binding Constraint – Range in which the right-hand-side of the constraint may move while keeping the constraint non-binding.

Allowable Increase/Decrease of an objective coefficient – Range in which the objective coefficient may move without disrupting the optimal solution.

Shadow Price – The change in the value of the target cell for every one-unit increase of the RHS of a constraint.

Reduced Cost – The amount the objective coefficient must change before the non-negativity constraint of the given decision becomes non-binding.

Relative reference – A reference in the form A1 that will change in value when auto-filled to other cells.

Absolute Reference – A reference in the form A1 that will not change in value when auto-filled to other cells.

Solving Algebraically and Graphically

Problem:

You are a student running a business selling two combos of Pocky: Combo A earns you $5, consisting of one chocolate and one strawberry, while Combo B earns you $6, consisting of two chocolates and no strawberry. You have 10 chocolates in stock and 20 strawberries in stock. Assume there are no costs associated with this model.

1. What is the objective function? Is this a maximizing or minimizing model?

2. List out all constraints in algebraic form. How many constraints are there?

3. Draw out the constraints and label the optimal solution on the provided graph template. What is the optimal solution, and what is the profit at that optimal solution? What are the binding constraints?

4. Find the allowable increase and decrease of the coefficients for both Combo A and Combo B. (Do this by hand.)

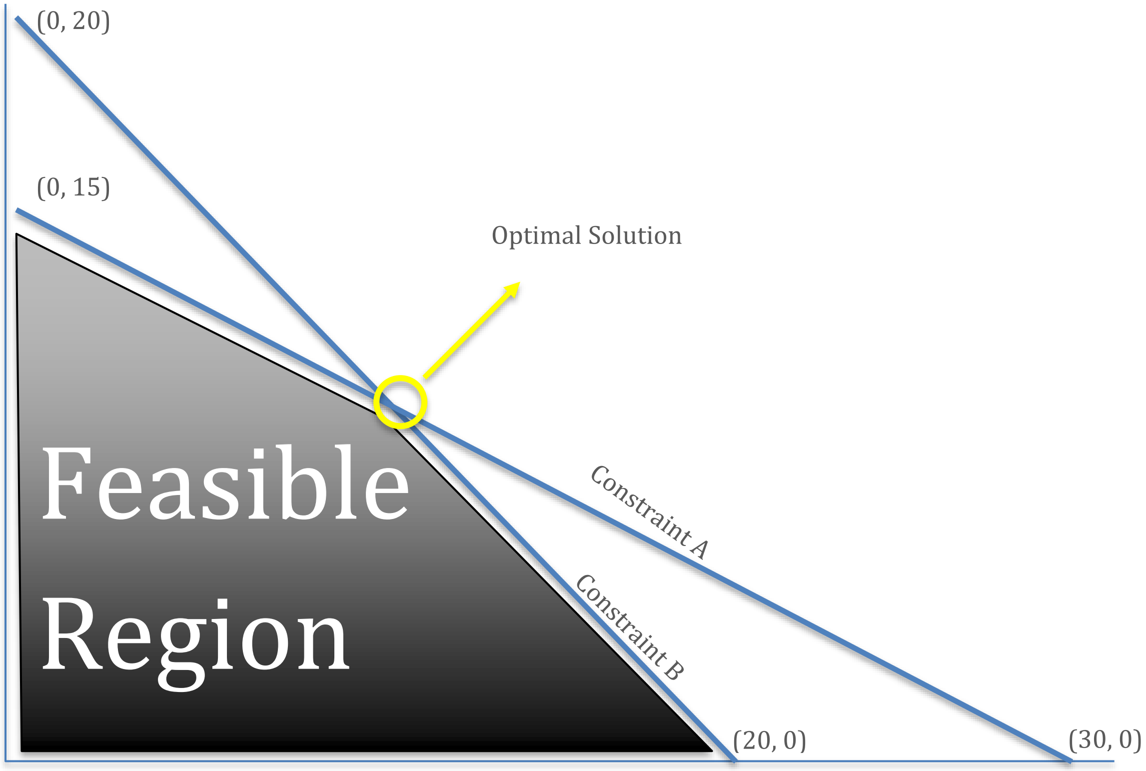
5. Find the allowable increase and decrease of all constraints. (Do this in Excel).

6. Suppose the price of Combo B increases to $10 while the price of Combo A increases to $7 due to a sudden influx in demand for chocolate Pocky. What is the new optimal solution? What is the new profit earned?

UNDERSTANDING GRAPHS

Problem:

Consider the following graph of an arbitrary linear programming model with the correct labelled optimal solution, feasible region. Assume this is a profit maximization model (That is, objective function is in the form xA + yB).

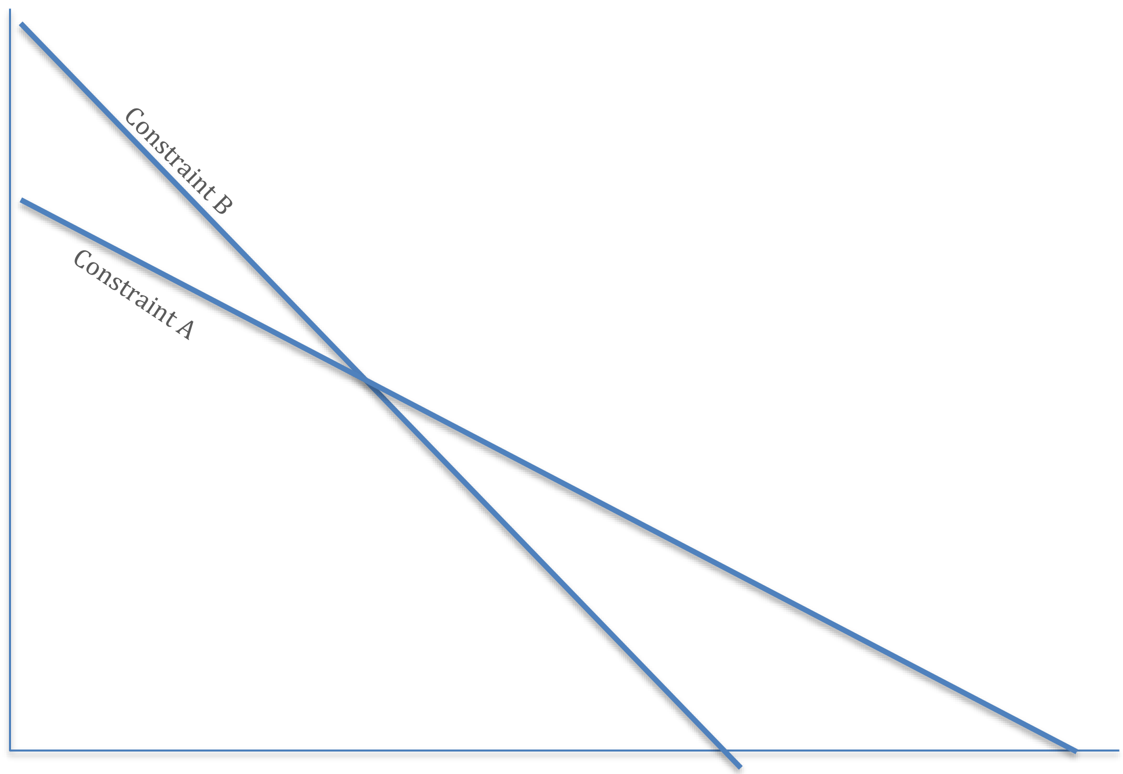


1. Define the range of possible slopes for isoprofit lines that would lead to this optimal solution. Give an example of a possible objective function.

2. Write two possible objective functions which would lead to multiple optima.

3. Find the coordinates of the optimal solution.

4. Suppose the sign of Constraint B (<=) is changed to the >= sign. How will this change the feasible region? Assuming the objective function remains the same and the LP remains feasible, draw in the new feasible region and label the new optimal solution. If this LP becomes infeasible, explain why.



FRUIT JUICE – EXCEL & SENSITIVITY ANALYSIS

Problem (Solve in Excel):

You are a company producing fruit juice producing four types of fruit juice: apple, orange, pineapple and tropical. You have 20L of apple concentrate, 20L of orange concentrate and 10L of pineapple concentrate. You have 60L of water. The breakdown of each juice is as follows:

- **One bottle of apple juice requires 200mL of apple concentrate and 300mL water;**
- **One bottle of orange juice requires 300mL of orange concentrate and 200mL water;**
- **One bottle of pineapple juice requires 250mL of pineapple concentrate and 250mL water;**
- **One bottle of tropical juice requires 150mL of pineapple, 100mL of orange, 50mL of apple and 200mL water.**

You profit $3 for every bottle of apple juice, $3 for every bottle of orange juice, $6 for every bottle of pineapple juice and $5 for every bottle of tropical juice.

1. Complete the algebraic formulation for this problem. You do **not** have to solve for optimal solution algebraically.

2. Solve for optimal solution in excel. What is the amount of profit made under the optimal solution? How many constraints are there? What about binding constraints?

3. Under the optimal solution, how many bottle of each fruit juice will you produce?'

4. Consider the following sensitivity analysis for Fruit Juice and answer the following questions.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Bottles to Make Apple	[redacted]	0	3	1E+30	1.4
D14	Bottles to Make Orange	[redacted]	0	3	1E+30	1.05
E14	Bottles to Make Pineapple	[redacted]	0	6	1E+30	0.583333333
F14	Bottles to Make Tropical	[redacted]	-0.35	5	0.35	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G5	Apple Concentrate Total	20	15	20	4.444444444	20
G6	Orange Concentrate Total	20	10	20	10	20
G7	Pineapple Concentrate Total	10	24	10	6.666666667	10
G8	Water Total	53.33333333	0	60	1E+30	6.666666667

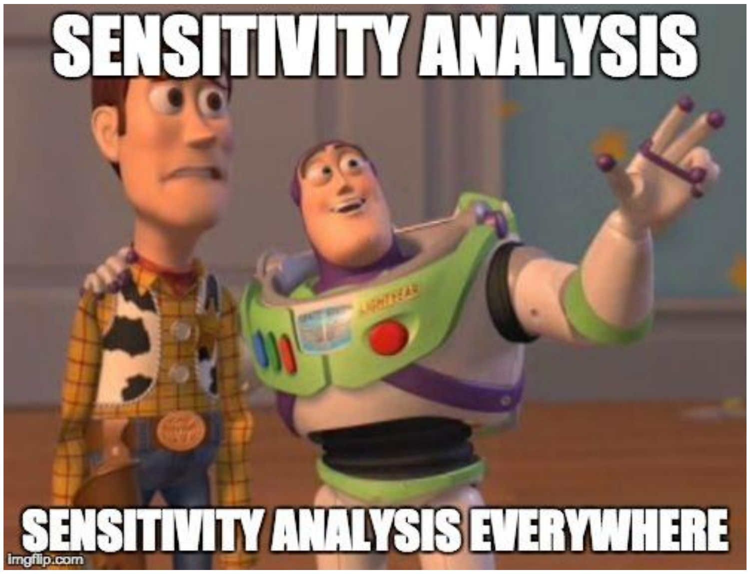
a. Let's say a sudden decrease in demand for tropical fruit juice drops the price to $3 per bottle. Will that change the optimal solution? Why or why not?

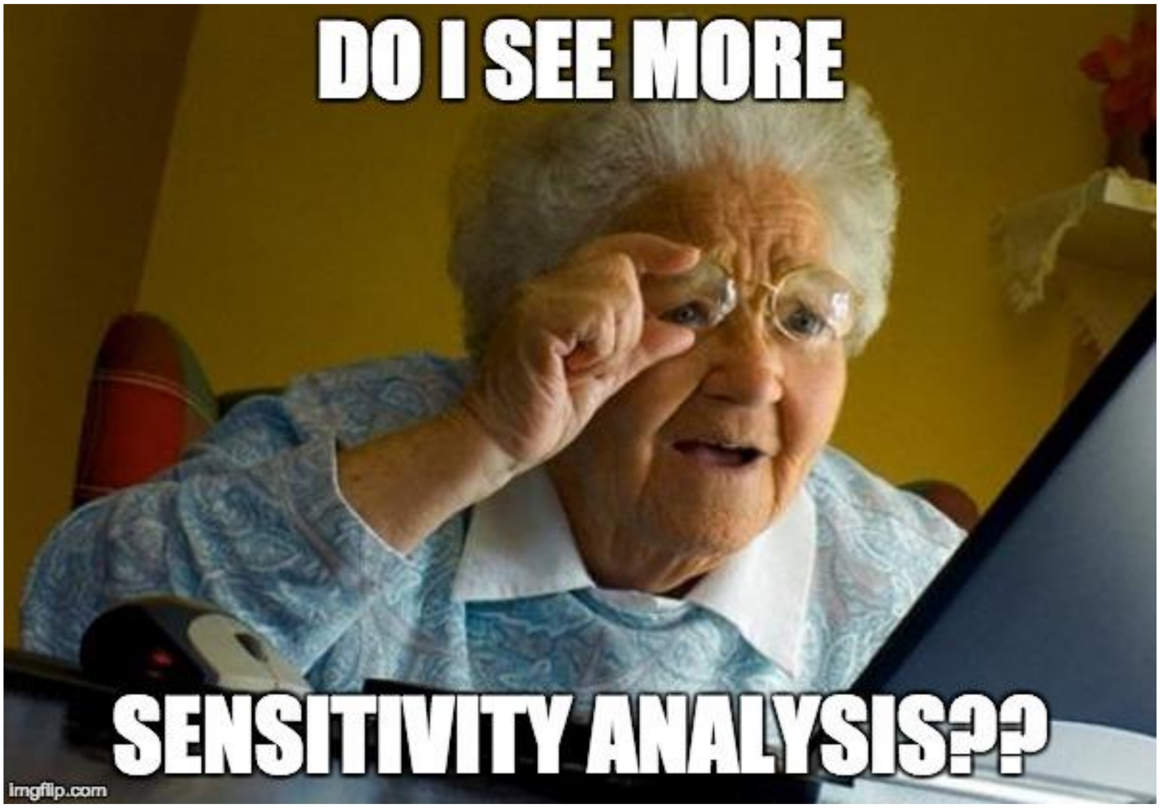
b. A drought has occurred and the supply of water has suddenly dropped from 60 litres to 45 litres. Will the optimal solution change? If yes, what is the new optimal solution, and how much profit will you make??

c. Suppose due to high demand, you must make at least 40 bottles of tropical juice. What is the new optimal solution, and how much profit will you make? Is this new constraint binding?

More On Sensitivity Analysis

Yes, I know what you're thinking…….





Problem:

Consider the following sensitivity analysis for an unspecified LP model. Some cells have had their numbers removed. Cells with a number attached and highlighted will be referred to in the questions in this section.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C16	Decision 1	4500	0	0.33	0.01	0.016
D16	Decision 2	0	0	0.33	1E+30	1E+30
E16	Decision 3	62000	0	0.2	1E+30	0.017
F16	Decision 4	0	-0.062	-8.81E-10	0.03	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C25	Constraint 1	(1)	0.65	5400	2501	1008
C26	Constraint 2	[redacted]	0.55	21600	6500	5300
C27	Constraint 3	[redacted]	0.14	0	12000	(2)
C29	Constraint 4	[redacted]	0.62	7500	28000	7500
C28	Constraint 5	62000	0.062	(3)	22000	27000

1. What should be the value within the cell (1)? Explain.

2. What should be the value within the cell (2)? Explain.

3. What should be the value within the cell (3)? Explain.

4. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.01?

5. How much more profit would the firm earn if Decision 2's objective coefficient went up by 0.5?

6. How much more profit would the firm earn if Decision 1's objective coefficient went up by 0.1 and Decision 4's objective coefficient went up by 0.02?

7. How much more profit would the firm make if the RHS of Constraint 1 was increased by 2500?

8. How much more profit would the firm make if the RHS of Constraint 3 was increased by 1000?

Computer Parts - Blending Problem

Problem:

You are the owner of a store for junk computer parts. You have CPUs, RAMs, and SSDs, which you can supply at the cost of $14.4, $12 and $9 each, respectively. You offer two types of blends: basic and premium. For basic, you will charge $33 for each piece of hardware, while for premium you will charge $36 per piece of hardware. You have 420 CPUs, 350 RAMs and 210 SSDs in inventory. However, there are a few guidelines you must follow:

- **Basic must contain:**
  - **At least 30% SSDs;**
  - **At most 50% RAMs;**
  - **At least 30% CPUs;**
- **Premium must contain:**
  - **At most 40% SSDs;**
  - **At least 35% RAMs;**
  - **At most 40% CPUs.**

1. Consider the following partially completed spreadsheet. There are some cells highlighted in blue which do not have their values filled in. What should be the best formula for each of the following cells labelled (a) to (g)?

Computer Parts							
Input Data							
		CPU	RAMs	SSDs			Price per piece
	Cost ($)	$ 14.40	$ 12.00	$ 9.00		Basic	$ 33.00
						Premium	$ 36.00
Action Plan							
		CPUs	RAMs	SSDs	Total		
	Basic				0		
	Premium				(a)		
	Total	(b)	0	0			
		<=	<=	<=			
	Constraint	200	300	400			
Blending Constraints							
					Output		Constraint
	Basic must be at least		30%	SSD	(c)	>=	(d)
	Basic must be at most		50%	RAM	0	<=	0
	Basic must be at least		30%	CPU	0	>=	0
	Premium must be at most		40%	SSD	0	<=	(e)
	Premium must be at least		35%	RAM	0	>=	0
	Premium must be at most		40%	CPU	0	<=	0
Revenue/Cost							
		CPUs	RAMs	SSDs			
	Basic	$ -	(f)	$ -	Revenue	Profit	
	Ultra	$ -	$ -	$ -	(g)	$ -	

a. Use this space to complete Problem 1.

2. How much of each bulk should you produce to maximize profit? What is the breakdown of each part among each blend?

3. Suppose you are doing an algebraic formulation for this blending problem. Write down all the blending constraints in algebraic form. Use the following labels: BC, BR, BS, PC, PR, PS, with the first letter representing the blend and the second letter representing the part.

4. Consider the following sensitivity analysis for Computer Parts with the optimal solution blacked out and answer the following questions. Try not to look at your completed optimal solution when solving these problems.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C9	Basic CPUs	[redacted]	0	18.6	3	1E+30
D9	Basic RAMs	[redacted]	-3	21	3	1E+30
E9	Basic SSDs	[redacted]	0	24	3	24
C10	Premium CPUs	[redacted]	0	21.6	1E+30	3
D10	Premium RAMs	[redacted]	0	24	1E+30	3
E10	Premium SSDs	[redacted]	0	27	1E+30	3

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C11	Total CPUs	[redacted]	21.6	200	146.6666667	160
D11	Total RAMs	[redacted]	24	300	1E+30	31.66666667
E11	Total SSDs	[redacted]	24	400	1E+30	53.33333333
F17	SSD Output	[redacted]	0	40	53.33333333	1E+30
F18	RAM Output	[redacted]	0	66.66666667	1E+30	66.66666667
F19	CPU Output	[redacted]	-3	40	160	40
F20	SSD Output	[redacted]	3	306.6666667	53.33333333	306.6666667
F21	RAM Output	[redacted]	0	268.3333333	31.66666667	1E+30
F22	CPU Output	[redacted]	0	306.6666667	1E+30	146.6666667

a. How many constraints are **binding**? (Ignore non-negativity constraints)

b. Due to an increase in demand, the price of the basic blend has increased from $33 to $36. Will this change the optimal solution? If yes, by how much will this increase the value in the target cell? If not, why not?

FOOD SERVICES – SCHEDULING

Problem:

You are the manager of a 24-hour fast-food restaurant on campus. Your restaurant offers six labour shifts per 24-hour period, starting at 12am, 4am, 8am, 12pm, 4pm, 8pm, and 12pm. You have access to workers who work two shifts a day. Due to fluctuations in demand, your required labor at different time periods is as follows:

- **12am-4am: 3 workers**
- **4am-8am: 4 workers**
- **8am-12pm: 7 workers**
- **12pm-4pm: 8 workers**
- **4pm-8pm: 6 workers**
- **8pm-12am: 5 workers**

Find the scheduling method that will use the minimum amount of workers. Produce a sensitivity analysis.

1. Is this a maximizing or minimizing model? What are you trying to maximize or minimize?

2. What is the optimal solution? How many binding constraints are there? What about non-binding?

3. Due to an overnight frat party, your demand for workers at 12am-4am goes up to four. Will this affect your optimal solution? Why or why not? If yes, what is the new optimal solution?

    a. Suppose the demand for workers at 4am-8am also increased by one. How will this affect the optimal solution?

CANADA POST – TRANSPORTATION

Problem:

You are the manager of a few Canada post branches in Vancouver. You have three branches under your control: Robson, Pine and Oak, and you must deliver parcels to UBC, YVR Airport and Oakridge center. Assume all parcels are identical. The supply and demand at each location is as follows:

- **Robson holds 7 parcels, Pine holds 16 and Oak holds 13.**
- **UBC requires 17, YVR Airport requires 5 and Oakridge requires 14.**
- **The shipping costs are as follows:**

		Deliver To:		
		UBC	YVR Airport	Oakridge
Deliver From:	Robson	$4.00	$4.50	$2.50
	Pine	$3.00	$4.00	$2.50
	Oak	$3.50	$3.00	$2.00

Solve for the optimal solution, and produce a sensitivity analysis.

1. What is the optimal solution, and how many constraints are binding?

2. What will happen to the LP if suddenly, an explosion happens at the Robson branch and three of the seven parcels in stock are destroyed?

3. What are the objective coefficients in this model?

4. **Refer to the produced sensitivity analysis for the next questions. If you cannot answer the question without running Solver, please answer "we don't know for sure."**
   a. Is there evidence of multiple optima in this LP?

   b. Suppose due to the acquisition of a new truck, it now costs $2.50 to deliver from Oak to YVR airport. Will this change the optimal solution? What will be the new value in the target cell?

   c. Suppose Pine has increased their supply of parcels by one. How will this affect the target cell under the optimal solution?

   d. Suppose Pine increases their supply of parcels by five to a total of 21. What will be the new amount of parcels shipped from Pine?

   e. Suppose Oak increases their supply of parcels by five to a total of 18. What will be the new target cell value under the new optimal solution? '

   f. Due to new hires, the cost of all shipments from Oak have been reduced by $0.10. What will be the new optimal solution?

   g. Shipments from Pine to UBC are now free. What is the new optimal solution and target cell value?