MATH 104/184

MIDTERM REVIEW SESSION

BY RAYMOND SITU

Table of Content

Things you should know.

$$\frac{d}{dx}\left(\sin(x)\right)=\cos(x) \qquad \frac{d}{dx}\left(\cos(x)\right)=-\sin(x)$$

$$\frac{d}{dx}\left(\tan(x)\right)=\sec^2(x) \qquad \frac{d}{dx}\left(\cot(x)\right)=-\csc^2(x)$$

$$\frac{d}{dx}\left(\sec(x)\right)=\sec(x)\tan(x) \qquad \frac{d}{dx}\left(\csc(x)\right)=-\csc(x)\cot(x)$$

θ	0 0	$\frac{\pi}{6}$ 30°	$\frac{\pi}{4}$ 45°	$\frac{\pi}{3}$ 60°	$\frac{\pi}{2}$ 90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	U
$\csc(\theta)$	U	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec(\theta)$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	U
$\cot(\theta)$	U	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b(x^n) = n\log_b x$$

$$\log_b(b) = 1$$

$$\log_b(1) = 0$$

I. RELATED RATES

- Application of implicit differentiation
- Taking derivative with respect to time
- Draw a diagram and label which variables are constant and which are changing
- Find an equation that ties the variables together
- Find the rate of change that is needed
- Remember to use the correct units

1. A conical pool has a radius of 3m and a height of 6. The pool drain is opened at the bottom and it is losing water at 2 cubic meter per minute. How fast is the water level dropping when the water is exactly 4m deep?

2. A conical pool has a radius of 3m and a height of 6. The pool drain is opened at the bottom and it is losing water at 2 cubic meter per minute. How fast is the radius shrinking by when the water is exactly 4m deep?

3. A kite is flying at an angle of elevation of $\pi/3$. The kite string is being pulled in at a rate of 1 meter per second. If the angle of elevation does not change, how fast is the kite losing altitude?

4. A movie is being filmed, the camera is located 40m from a straight highway and a car is traveling on it at 30m/s. The camera turns so it is always pointed at the car. How fast is the camera's angle changing when the car is 40m away from the camera?

II. Applications of Calculus

1. Differentiate y = x^x, state your answer using only x

2. Differentiate $y = \frac{(x^2+1)^2(x^3+2)^4}{(x+3)^8}$, you do not need to simplify your answer

III. PRICE ELASTICITY OF DEMAND

$E = \frac{\%\Delta Q}{\%\Delta P} = \frac{dQ/Q}{dP/P} = \frac{P}{Q}\frac{dQ}{dP}$ This is where the formula comes from.

When |E|>1 we say it is price elastic

When |E|<1 we say it is price inelastic

When |E|=1 we say the price is unit elastic, this is the optimal solution that maximizes revenue

1. Air Canada has determined that the demand for its one way tickets from LAX to YVR is: $lnq - lnp + 0.01 = 21$. q is the number of tickets and p is the price in dollars per ticket. What price should they set it at to maximize revenue?

2. Now find another way to do question 1

IV. First and Second Derivative Tests

For questions asking for ABSOLUTE max or min.

1.Take derivative

2. Find points where the derivative is equal to 0 or undefined
3. Plug and chug each of those points AND the boundary points for the **y value**
4. Pick out the max and min

For questions asking for LOCAL max or min

1.Take derivative

2. Find points where the derivative is equal to 0 or undefined
3. Draw number line and find the **SIGN** of the **first derivative** of the intervals.

    Negative means decreasing, positive means increasing.

    if it goes from increasing to decreasing it is a local max.

    if it goes from decreasing to increasing it is a local min.

OR plug and chug the **SIGN** of the **second derivative** of the all the points.

    Negative means concave down and it is a local max

    Positive means concave up and it is a local min

1. Find the absolute minimum of f(x) = $\frac{2x}{\sqrt{2x-3}}$ on the interval [2,6]

2. List out all the critical points of $f(x) = \frac{x^2-1}{x^3}$, list out the increasing and decreasing intervals, list out the local max and min

V. CURVE SKETCHING

What to do:

1. Domain
2. Asymptotes
3. x and y intercept
4. Intervals of increase/decrease
5. Local max/min
6. Concavity
7. Draw!

1. Sketch the curve $f(x) = 3x^5 - 5x^3$