COMM 295

Final Review Session

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Table of Content

Repeated Games

1. Consider a game with the following payoff table:

Player 2

		Cheat	Cooperate
Player 1	Cheat	5 5	-20 50
	Cooperate	50 -20	20 20

a. What is the Nash equilibrium(a) if the game is only played once? Why?

If Player 1 cheats, Player 2's best response is to cheat. If Player 1 cooperates, Player 2's best response is to cheat. If Player 2 cheats, Player 1's best response is to cheat. If Player 2 cooperates, Player 1's best response is to cheat. Therefore, the Nash equilibrium is when both players cheat and each of them gains 5.

b. What is the Nash equilibrium(a) if the game is played indefinitely with a tit-for-tat strategy in place? Why?

Tit-for-tat strategy refers to a repeated game strategy where both players agree to cooperate in the beginning, and a player takes the action that its rival took in the previous round (e.g. if Player 1 cheats this round, Player2 would cheat in the next round) to retaliate for the deviation from their mutual agreement.

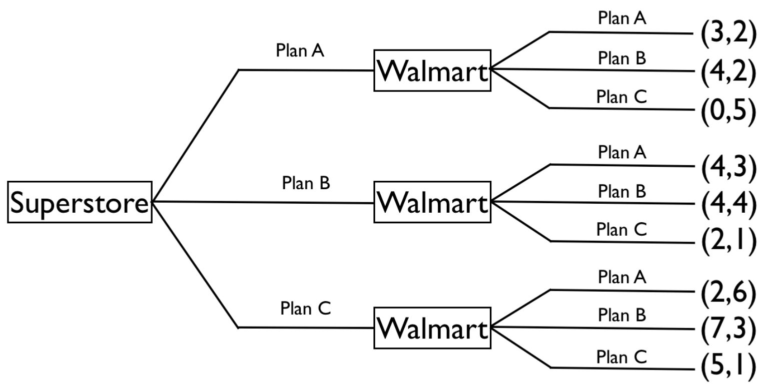
If both players cooperate, each of them makes 20. If, for example, Player 1 cheats, when Player 2 cooperates, Player 1 makes 50 while Player 2 loses 20. Player 1 makes 50-20=30 more this period from cheating. However, with the tit-for-tat strategy in place, Player 2 will cheat the next round for retaliation.

Player 1 will need to cooperate again the next round for the cooperation to continue. So Player 1 will make -20 while Player 2 makes 50 in the next round. The net profit of Player 1 from cheating is 50 (from cheating)-20 (profit from cooperating)-20 (profit from cooperating)-20 (profit from cooperating again with the tit-for-tat strategy played) = -10. There is a net loss of 10 if Player 1 cheats for one period. Therefore, both players do not have the incentive to cheat if the game is expected to continue indefinitely. The Nash equilibrium will be (20,20) where both players cooperate as a result.

c. What is the Nash equilibrium(a) if the game lasts two periods? Three periods? Why?

   In a finite repeated game—and given that the number of games is common knowledge to both players—the last period of the game is like a static prison dilemma; the best response for each player is to cheat. Player 1 knows that Player 2 will cheat in the last period because the game no longer repeats after the last period. Player 1 knows that Player 2 will cheat in the last period for the same reason. Both players know they are both going to cheat in the last period for certain. Given this knowledge, they cannot play a cooperative strategy at (20,20) and have incentive to cheat in the first period as well. The same reasoning applies for a finite repeated game that lasts three periods. Any finite game unravels itself and the Nash equilibrium would be (5,5) where both players cheat to optimize their payoffs.

2. Superstore and Walmart are the two largest supermarket companies in a local city. Suppose Superstore is the first mover to pick a product pricing plan, so which plan Walmart picks is contingent upon Superstore's move. Which plans will Superstore and Walmart end up picking?



Since Superstore gets to move first, it can choose a plan based on what it expects Walmart to pick contingent upon Superstore's strategy. Using backward induction, Superstore knows that Walmart will pick plan C to maximize payoff when Superstore picks plan A, so the expected payoff for Superstore and Walmart would be (0,5) where Superstore makes 0 and Walmart makes 5 if Superstore picks plan A. Superstore knows that when it picks plan B, Walmart will pick plan B to maximize its payoff. So the expected payoff for Superstore and Walmart would be (4,4) where both of them make 4. Superstore knows that when it picks plan C, Walmart will pick plan A to maximize payoff. So the expected payoff for Superstore and Walmart would be (2,6), where Superstore makes 2 and Walmart makes 6.

With all the possible combinations of payoff—(0,5) when Superstore picks plan A, (4,4) when Superstore picks plan B and (2,6) when Superstore picks plan C—Superstore pick the plan that maximizes its own payoff. So, Superstore will pick plan A where it makes a profit of 4. Walmart will pick plan B as a result.

3. Suppose that Superstore and Walmart pick their product pricing plans simultaneously every month. Superstore has the option to pay $1.5 million (equivalent to 1.5 on the payoff table) for the right to move first.

		Walmart		
		Plan A	Plan B	Plan C
Superstore	Plan A	3 6	2 5	0 4
	Plan B	2 0	4 4	2 3
	Plan C	1 4	0 3	5 5

a. What is the Nash equilibrium(a)?

If Superstore picks plan A, Walmart's best response would be plan A.

If Superstore picks plan B, Walmart's best response would be plan B.

If Superstore picks plan C, Walmart's best response would be plan C.

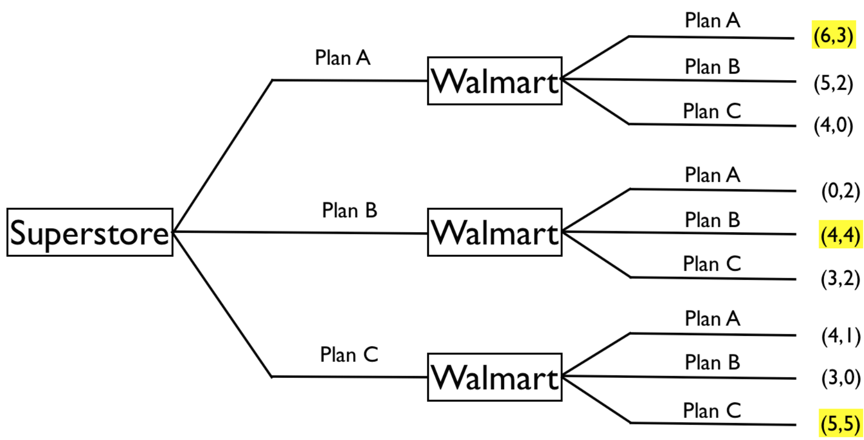
If Walmart picks plan A, Superstore's best response would be plan A.

If Walmart picks plan B, Superstore's best response would be plan A.

If Walmart picks plan C, Superstore's best response would be plan C.

There are two Nash equilibria (6,3) where both pick plan A, and (5,5) where both pick plan C.

b. What is the Stackelberg equilibrium if Superstore moves first assuming that it does not cost Superstore anything to be the first mover?



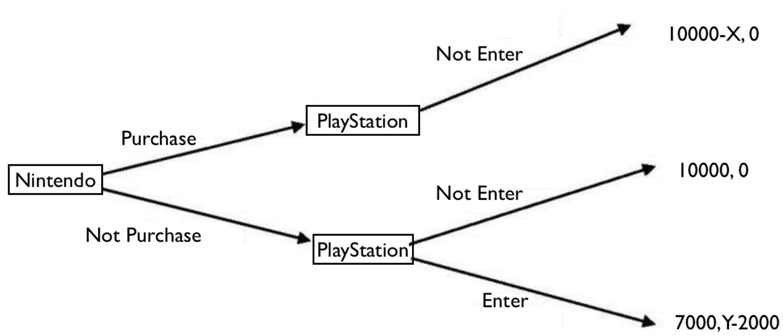
Using backward induction, Superstore knows that Walmart will pick plan A to maximize payoff when Superstore picks plan A, so the expected payoff for Superstore and Walmart would be (6,3) where Superstore makes 6 and Walmart makes 3. Superstore knows that when it picks plan B, Walmart will pick plan B to maximize its payoff. So the expected payoff for Superstore and Walmart would be (4,4) where both of them make 4. Superstore knows that when it picks plan C, Walmart will pick plan C to maximize payoff. So the expected payoff for Superstore and Walmart would be (5,5), where both of them make 5.

With all the possible combinations of payoff—(6,3) when Superstore picks plan A, (4,4) when Superstore picks plan B and (5,5) when Superstore picks plan C—Superstore picks the plan that maximizes its own payoff. So, Superstore will pick plan A where it makes a profit of 6. Walmart will pick plan A as a result.

c. Should Superstore pay for the first-mover right?

If Superstore pays $1.5 million for the right to move first, it is directly subtracting 1.5 from any payoff it makes. So if Superstore pays 1.5 for the right to pick plan A where Superstore expects to make 6, Superstore will end up making only 6-1.5=4.5, which is lower than the payoff of 5 if Superstore and Walmart pre-communicate to simultaneously pick plan C, where both firms yield a payoff of (5,5). It is not worth the paying $1.5 million for the first mover right to pick plan A.

4. Nintendo has long been a success in releasing a series of Pokémon video games. PlayStation considers introducing a series of Pokémon video games as well. Such move will dilute Nintendo's market share. However, Nintendo has the option of purchasing the Pokémon Company and monopolizing the Pokémon franchise to prevent competitors from making Pokémon video games.



a. Assume that X stands for is the amount that Nintendo can pay for to keep PlayStation from entering the market. At what cost will Nintendo purchase Pokémon Company to keep PlayStation out of the competition?

10000-X >/= 7000

10000-7000 </= X

X </= 3000

Nintendo is willing to pay up to 3000 to deter PlayStation from entering the market. If Nintendo needs to pay anything above 3000 to prevent entry, Nintendo will make a profit of <7000, which is lower than what it would make if PlayStation enters the market. In that case, Nintendo would rather accommodate entry.

b. Assume that Y stands for the profit that PlayStation expects to make upon entering the market. At what profit does PlayStation need to make to enter the market of Pokémon video game?

The 2000 here stands for the fixed cost (upfront cost) that PlayStation needs to pay to enter the market

Y-2000>0

Y>2000

If Y </= 2000, PlayStation will be making 0 or negative profit. In that case, PlayStation would rather stay out of the market. Therefore, PlayStation must make at least $2000 to be willing to enter the market.

c. Suppose competitors do not have complete information of Nintendo's profits, will Nintendo pay a cost of 4000 to monopolize the Pokémon franchise? What if there will be more than just PlayStation considering entering the Pokémon video game market?

Nintendo will not spend 4000 to deter PlayStation from entering because it will make a profit of 10000-4000=6000, which is lower than the 7000 that it would be making if PlayStation enters the market. However, if there will be more than just PlayStations considering entry, Nintendo will spend the 4000 at the cost of its own to fight back and to build reputation. Fighting PlayStation will be a rational long-run strategy that Nintendo may consider in deterring other competitors from entering as well.

BEHAVIORAL DECISION-MAKING UNDER UNCERTAINTY

Kelvin studies Finance in a business school and believes he can make wise investment decisions. He has some savings and would like to spend all the money on investment. He is faced with three investment options:

1) 50% chance of earning $200 in return and 50% chance of earning only $20 in return
2) 35% chance of earning $250 in return and 65% chance of earning only $10 in return
3) Lend the money to a family friend who promised to pay back $100 in addition to the initial amount borrowed

1. Which option has the highest risk?

   The formula for variance is given by σ^2=∑Pr[(X-E(X)^2)
   We need to calculate the expected value E(X) for each option first
   Option 1 E(X)=0.5*200+0.5*20=110
   Option 2 E(X)=0.35*250+0.65*10=94
   Option 3 E(X)=100
   Option 1 σ^2=0.5*(200-110)^2+0.5*(20-110)^2=8100
   Option 2 σ^2=0.35*(250-94)^2+0.65*(10-94)^2=13104
   Option 3 σ^2=0
   The option with the highest variance is the riskiest—option 2

2. Which investment option will Kelvin pick if Kelvin is…

   Compare the expected utilities

   a. **Risk neutral with utility function U(x)=x?**

EU of option 1=0.5*(200)+0.5*(20)=110

EU of option 2= 0.35*(250)+0.65*(10)=94

EU of option 3=100

Option 1 gives the highest utility (110>100>94), therefore risk neutral individuals would prefer option 1.

b. Risk averse with utility function $U(x)=x^{1/2}$?

EU of option 1=$0.5*(200)^{1/2}+0.5*(20)^{1/2}=9.31$

EU of option 2= $0.35*(250)^{1/2}+0.65*(10)^{1/2}=7.59$

EU of option 3=$(100)^{1/2}=10$

Option 3 gives the highest utility (10>9.31>7.59), therefore risk-averse individuals would prefer option 3.

c. Risk-preferring with utility function $U(x)=x^2$?

EU of option 1=$0.5*(200)^2+0.5*(20)^2=20200$

EU of option 2= $0.35*(250)^2+0.65*(10)^2=21940$

EU of option 3=$(100)^2$=10000

Option 2 gives the highest utility (21940>20200>10000), therefore risk-averse individuals would prefer option 2.

3. Debbie is completing a gambling task for a Psychology experiment. She is given two options: she could either walk out of the room with $50, or enter a draw to have a 40% chance of winning $100 and a 60% chance of receiving only $20. Debbie's utility is defined by the utility function $U(x)=x^{0.25}$, where x is the amount of money she receives from the experiment.

    a. What is Debbie's expected utility from getting the $50?

       Given the utility function of $U(x)=x^{0.25}$ and that x=50 for the first option, derive the expected utility by plugging in x=50 into the function.

       U($50)=$(50)^{0.25}$=2.66

    b. What is Debbie's expected utility from entering the draw?

       Given the utility function of $U(x)=x^{0.25}$ and that there is 40% chance of winning $100 and 60% chance of winning $20 for the second option, derive the expected utility using the same method you would use to derive expected values.

       $EU=0.4*100^{0.25}+0.6*20^{0.25}=2.53$

    c. According to the expected utility hypothesis, which option will Debbie pick?

The expected utility hypothesis states that people act to maximize expected utility, therefore according to the hypothesis, Debbie would pick the option of walking away with the $50 over the option of entering the draw because walking away with $50 gives Debbie a higher expected utility (2.66>2.53).

d. What is the % chance of winning $100 in the draw that would induce Debbie to be indifferent between walking away with the $50 and entering the draw?

Set x=the chance of winning $100 and (1-x)=the chance of winning $20 in the draw, then equate the expected utility of entering the draw with the expected utility of the $50 option, which is 2.66.

$x*100^{0.25}+(1-x)*20^{0.25}=2.66$
$100^{0.25}x+20^{0.25}-20^{0.25}x=2.66$
$100^{0.25}x-20^{0.25}x=-20^{0.25}+2.66$
$1.047535133x=0.545257473$
$x=0.52 \rightarrow 52\%$

There must be around 52% chance of winning $100 in the draw for Debbie to be indifferent between the two options.

ADVERSE SELECTION

In a market of used gaming consoles, both good used gaming consoles and inferior used gaming consoles are available. Owners of those gaming consoles have information of the actual qualities of the gaming consoles, whereas the buyers do not. In the market, 50% of all used 3DS are good, and 50% are inferior. All buyers are risk neutral and are willing to pay $130 for a good used 3DS, but only $80 for an inferior used 3DS. The owners of the good used 3DS are willing to sell them at a price no lower than $120. The owners of the inferior used 3DS are willing to sell them at a price no lower than $60.

What is the equilibrium price? Is there adverse selection?

Since buyers do not have information of the qualities of the used 3DS, they are only willing to pay a price that is the expected value of a used 3DS (given that there is a 50% chance of potentially getting a good used 3DS and 50% chance of potentially getting an inferior used 3DS):

Expected value of used 3DS=0.5*80+0.5*130=$105

→ $105 is the price that a buyer is willing to pay for a used 3DS regardless of whether the used 3DS is good or inferior in actuality because the buyer cannot know the actual qualities of the 3DS

Sellers of inferior used 3DS are willing to accept a price no lower than $60, so they are willing to sell at the price of $105 ($105>$60)

Sellers of good used 3DS are willing to accept a price no lower than $120, so they are not willing to accept buyers' offer of $105. As a result, no good used 3DS gets sold.

There is an adverse selection as the market is dominated by the owners of the inferior used 3DS—buyer's expected value of $105 is less than the minimum amount that an owner of good used 3DS is willing to accept. Only inferior used 3DS will be sold. Realizing that there are only inferior 3DS left in the market, the equilibrium price will be $80, which is the price buyers are willing to pay for an inferior 3DS.

For what relative fractions of good used 3DS and inferior used 3DS will adverse selection not occur?

Adverse selection only occurs if information asymmetry causes the sellers of the good used 3DS to be out of the market, leaving only the sellers of inferior used 3DS in the market. Adverse selection won't occur if buyer's maximum willingness to pay (expected value) is at least or more than the reservation price of the good used 3DS seller.

Set x to be fraction of good used 3DS available in the market and (1-x) to be the fraction of inferior used 3DS available in the market.

$x*130+(1-x)*80=120$

$130x+80-80x=120$

$40=50x$

$x=0.8$ → If there are at least 80% of good used 3DS in the gaming console market, then buyers will be willing to pay $120 for a used 3DS of ambiguous quality, meeting a good used 3DS seller's reservation price of $120.

MORAL HAZARD AND AGENCY

Peter operates a stand that sells kites in the park. During good days, more profits get generated. During bad days, business declines and profits decline. There is an equal chance for a good day as for a bad day. Peter hires and pays Susan to sell kites for him. Susan can decide what level of effort she puts into working—the higher the effort, the more profits get generated. But Susan incurs a cost of $48 for putting in high effort. The table below summarizes the profits that the kite business expects to make under different situations. Determine whether Susan prefer to input normal or high effort in each situation, and what net benefit would Peter receive.

	Bad Day	**Good Day**
Normal Effort	80	240
High Effort	240	400

1. Peter pays Susan a fixed wage of $60 regardless of efforts.

   With normal effort, Susan gets paid $60. With high effort, Susan still gets paid $60 but also incurs a cost of $48, so her net benefit would be only $60-$48=$12 for putting in high effort. Susan would prefer normal effort.

   To derive what Peter expects to receive, find the expected return of operation under normal effort (note that there is 50% chance for a good day and 50% chance for a bad day):

   0.5*80+0.5*240=$160

   Then, take out the wage that Peter pays Susan:

   $160-$60=$100 → Peter expects to receive $100 in return

2. Peter could monitor Susan's performance and pay Susan a bonus of $40 for normal effort and $120 for high effort.

   With normal effort, Susan gets paid $40. With high effort, Susan gets paid $120 but incurs a cost of $48, so her net benefit would be $120-$48=$72 for putting in high effort. Susan perceives a higher benefit in putting in high effort ($72>$40), so would prefer high effort.

Since Susan is expected to put in high effort, Peter expects to receive an expected return under high effort (note that there is 50% chance for a good day and 50% chance for a bad day):

0.5*240+0.5*400=$320

Then, take out the wage that Peter pays Susan:

$320-$120=$200 → Peter expects to receive $200 in return

3. Peter offers Susan a 35% profit share

With normal effort, Susan gets paid:

0.5*0.35*80+0.5*0.35*240=$56

With high effort, Susan gets paid:

0.5*0.35*240+0.5*0.35*400-$48=$64

→ Susan would prefer high effort ($64>$56)

Since Susan is expected to put in high effort, Peter expects to receive an expected return under high effort. But Peter only receives 65% of the expected returns as 35% is paid to Susan as profit share:

0.5*0.65*240+0.5*0.65*400=$208 → Peter expects to receive $208 in return

4. What is the minimum profit share that Susan is willing to input high effort?

In other words, what % should profit share be to induce Susan's preference for high effort? Set x to be the % of profit share, and equate Susan's expected gain under normal effort and expected gain under high effort.

0.5*x*80+0.5*x*240=0.5*x*240+0.5*x*400-480

40x+120x=120x+200x-48

160x=320x-48

48=160x

x=0.3 → 30% is going to give Susan the same expected gain regardless of effort

At a profit share of 30%, Susan would likely prefer normal effort as exerting high effort gives the same expected gain as normal effort. Peter would need to pay a profit share greater than 30% to induce Susan's willingness to put in high effort

5. Which paying contract maximizes total surplus?

   Under fixed salary, the combined benefit of both Peter and Susan is:

   $60+$100=$160

   Under bonus contract, the combined benefit of both Peter and Susan is:

   $72+$200=$272

   Under profit sharing, the combined benefit of both Peter and Susan is:

   $64+$208=$272

   → Bonus contract and profit sharing both maximize total surplus (combined benefit of Peter and Susan)

MARKET FAILURE

1. Evaluate the following statements and identify whether each statement represents a Pareto improvement or Pareto efficiency.

    a. A cellular company priced a cellular plan at $50, but it decides to lower its price to $40, which is a price that would maximize total surplus.

       Pareto efficiency—even though the market efficiency has been maximized by eliminating deadweight loss and increasing consumer surplus, the cellular company has lowered its own surplus (producer surplus) to do so. The overall increase in efficiency is achieved at the loss of the cellular company.

    b. Peter has a burrito, but he prefers burger. Sally has a burger but prefers burrito. Peter and Sally decide to trade.

       Pareto improvement—both Peter and Sally are made better off with the trade taking place. Peter's surplus has increased by getting the burger and Sally's surplus has increased by getting the burrito.

    c. Assuming all lives have equal value. Molly donated her two kidneys to save the lives of two patients who needed kidney transplants.

       Pareto efficiency—If Molly did not sacrifice her kidneys, the two patients would die, so the total surplus would equate to one life. If Molly sacrifices her kidneys to save the two patients, the total surplus would equate to two lives assuming Molly could not survive without a kidney. Total surplus is improved at the cost of Molly's loss.

2. A plant producing gel for highlighter is in a perfectly competitive market. The gel is toxic and the production process involves pollution. The private marginal cost is given by MC=2Q and the external marginal cost is given by MC=0.25Q. If the market demand for this gel is Q=60-P, how much tax per unit of production should the government impose on the plant to induce efficiency?

Step 1. Find the output of the plant in this perfectly competitive market.
P=MC
Q=60-P → P=60-Q
Given that MC of the plant (private)=2Q, equate P=60-Q=MC=2Q
60-Q=2Q → Q=20

Step 2. Now that you have found the output level of the firm, which is Q=20, you substitute the output quantity into the external marginal cost function, MC=0.25Q, to find the external marginal cost per unit at the current output level.
MC=0.25*(20)
MC=$5

Step 3. Since the external marginal cost is $5 per unit at the current production level of Q=20, the government should impose a tax of $5 for every unit of gel produced, to induce efficiency.

3. A chemical plant that sells a harmful chemical has a cost function of C=30x+7 and can only sell the chemical at a price equivalent to its marginal cost per kg. The inverse market demand is represented by P=130-Q. The production of the chemical pollutes the nearby lake and affects those living around the plant. The negative effect incurs an external marginal cost of $10 per kg of the chemical produced. Calculate the deadweight loss induced by the externality.

Step 1. Find the market equilibrium price. Since market price is the plant's MC, derive MC from the cost function given.
MC=derivative of C=30x+7
MC=$30 → equilibrium price

Step 2. Find the equilibrium quantity by substituting P=$30 into the price function, which is given by P=130-Q.

$30=130-Q

Q=100

Step 3. Derive social marginal cost, which is equal to private marginal cost and external marginal cost.

$MC^S=MC^P+MC^E$

MC^S=$30+$10

MC^S=$40/kg

Step 4. Derive the socially efficient output level by substituting social marginal cost into the inverse market demand function of P=130-Q.

MC^S=$40=P=130-Q

$40=130-Q

Q=90

Step 5. Find the area of deadweight loss.

DWL=[($40-$30)*(110-100)]/2=50

→ Deadweight loss is 50