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MATH 104/184
Final Exam Review Session

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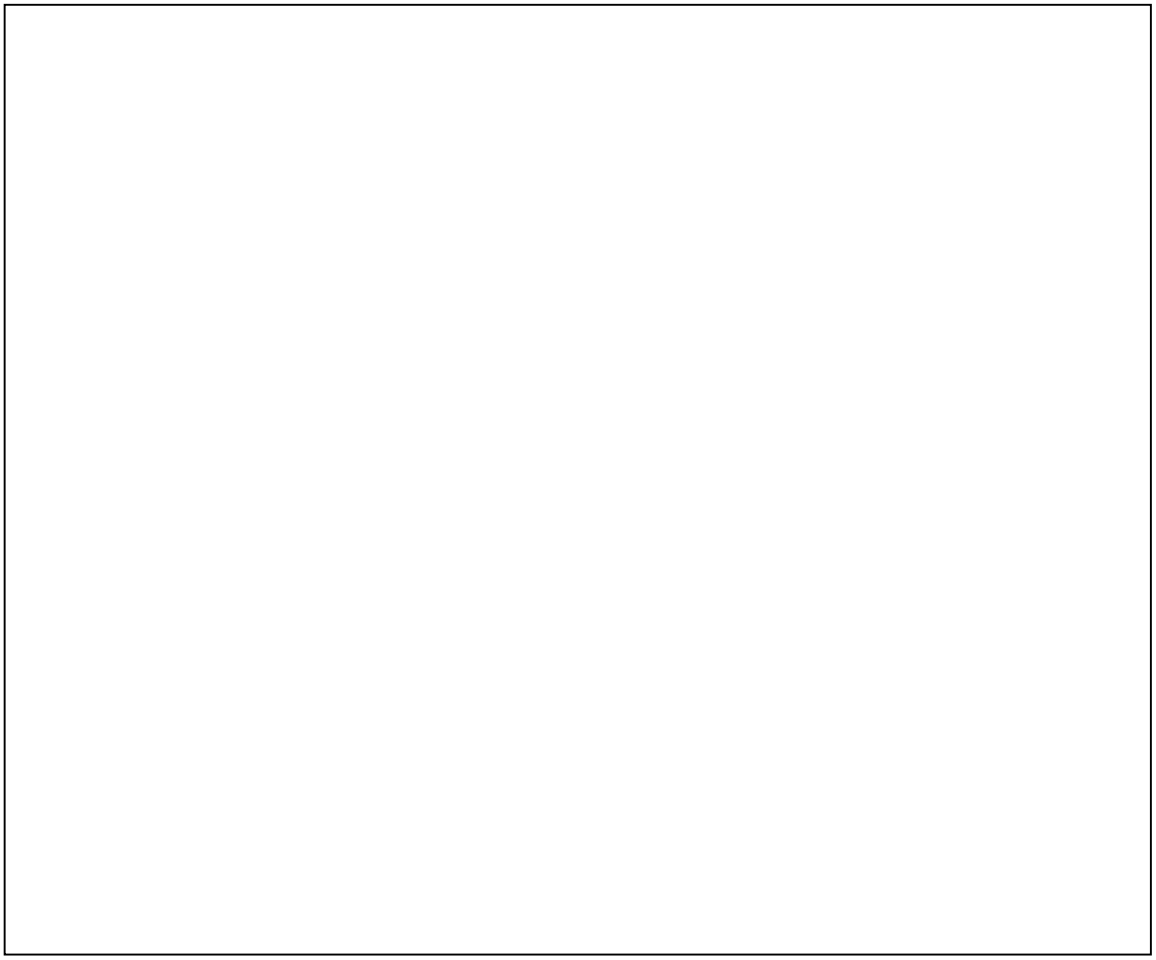
TABLE OF CONTENT

I. Related rates in 3 dimensions
II. Optimization
III. Local linear approximation
IV. Taylor polynomials
V. Curve sketching
VI. Some fun limits

Related rates

A plane is taking off the run way at a speed of 300 km/h due North. The angle of elevation is 30 degrees. A car is travelling due east at 150km/h on a straight, level, road. How fast is the distance between the plane and the car increasing when the plane has reached an altitude of 3km, assuming they both started from the same point and reached their respective velocities instantly. (red is plane, green is car, blue is distances). I heard there was a fun related rates on the midterm involving triangles so I made this questions with lots of triangles.



OPTIMIZATION

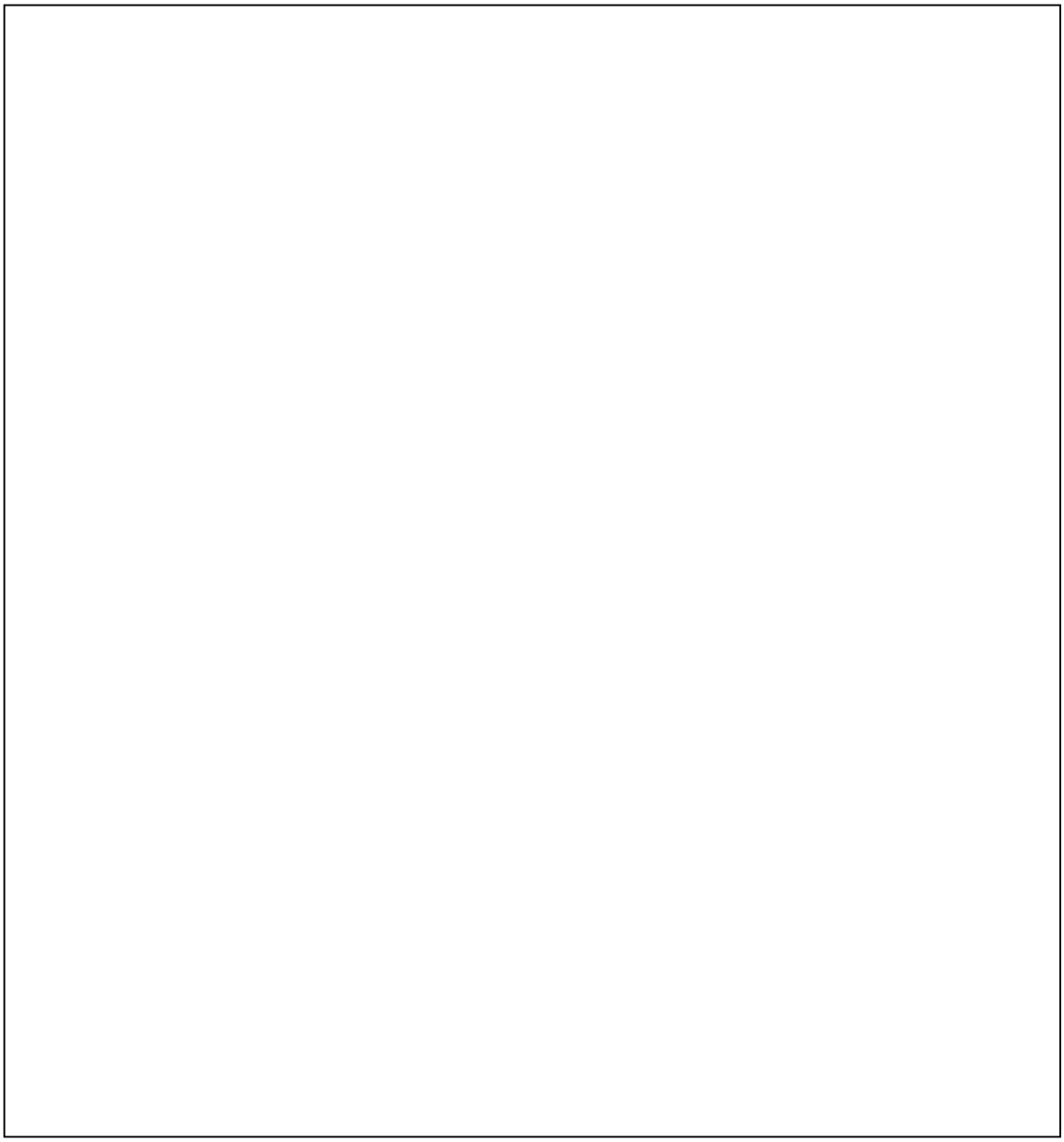
The first year students at UBC (University of Burnaby Coquitlam) have cried a river after failing their MATH 140/148 midterm. The river is 5km wide and the students need to get across to the other side to a point that is 10km downstream from the point directly across from their current position. The students can walk at a speed of 5km/h. The river is also so salty that the current is no longer flowing which allows the students to swim at a speed of 3km/h. What is the most efficient way for the students to *"get over it"*? (both physically and psychologically)

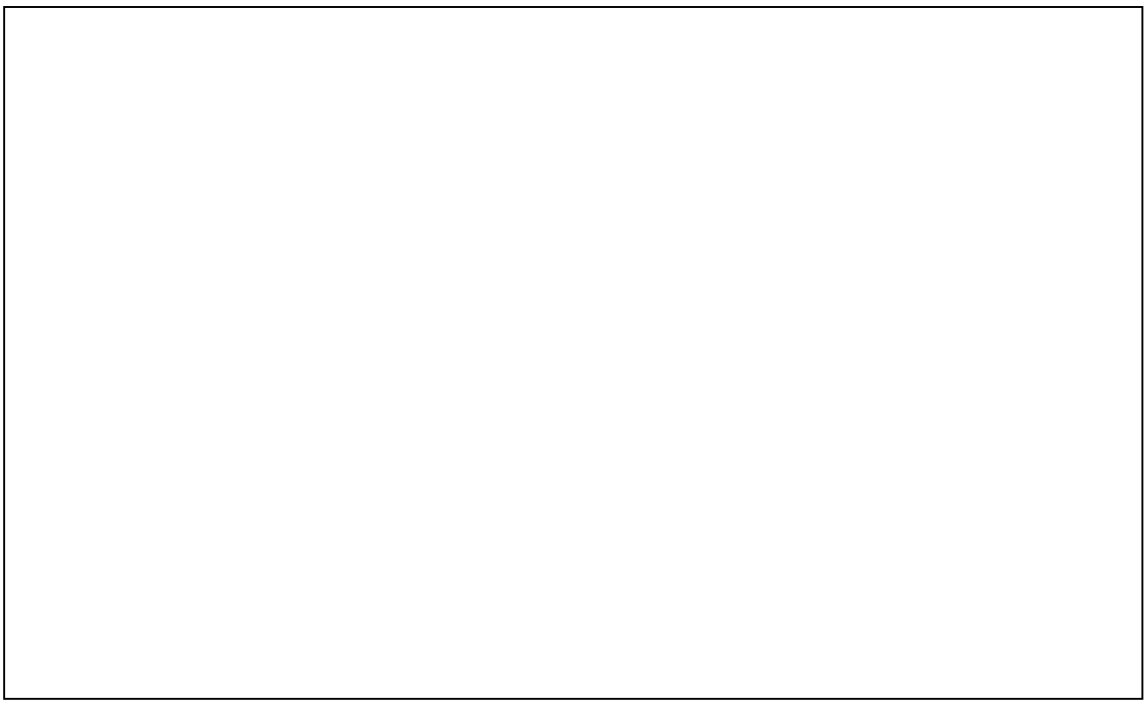
LOCAL LINEAR APPROXIMATION

Given: $\sqrt[3]{65}$

a) Find the linear approximation
b) Use the linear approximation to approximate the desired value
c) Determine if it is an overestimate or an underestimate
d) What is the bound on the error?

Originally, I had a very fun question planned for you guys but Professor Desaulniers said "I think this problem may confuse them" so I had to change it to a simpler one :(





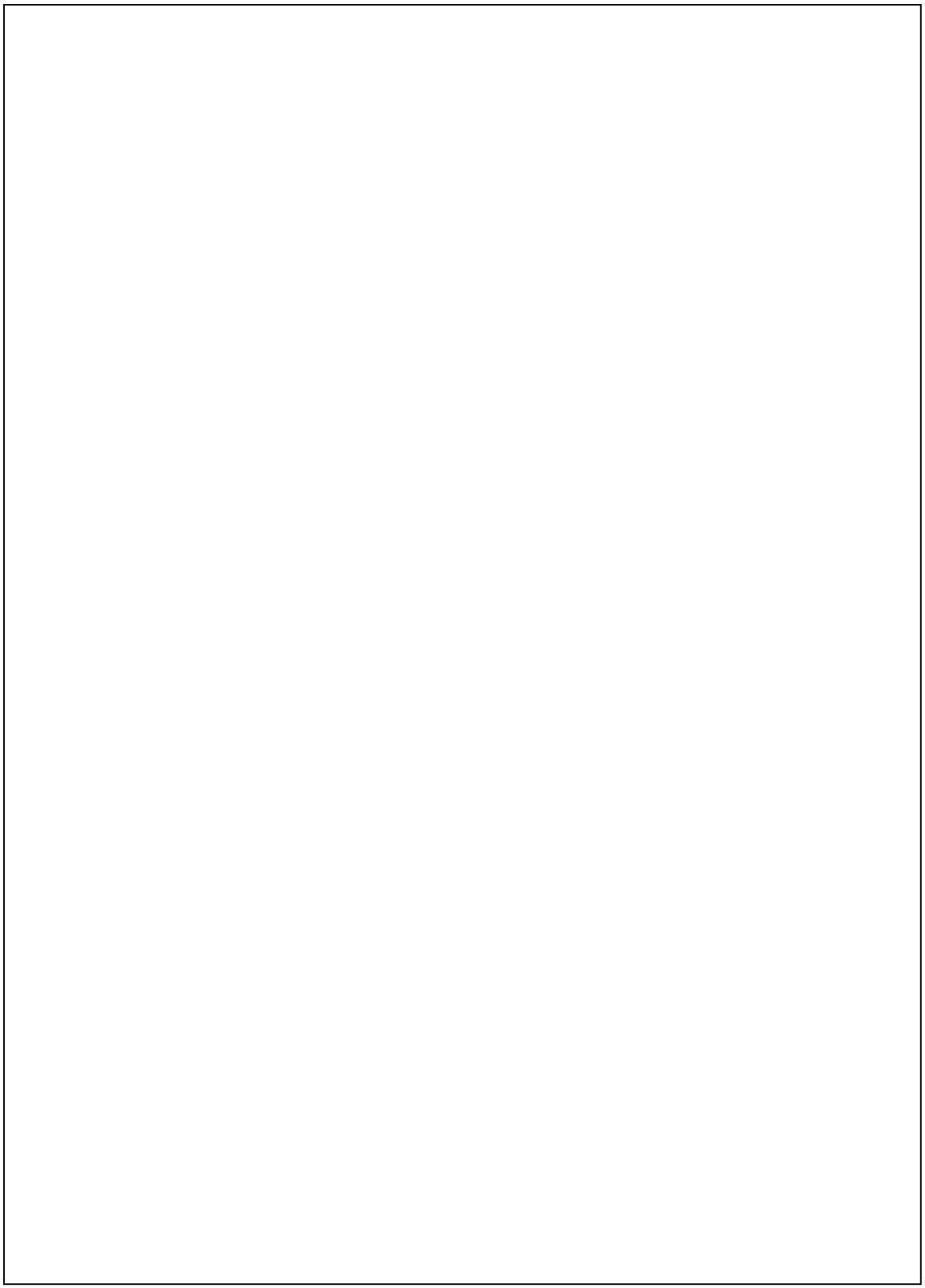
TAYLOR POLYNOMIALS

What is the 2nd degree Taylor polynomial of:

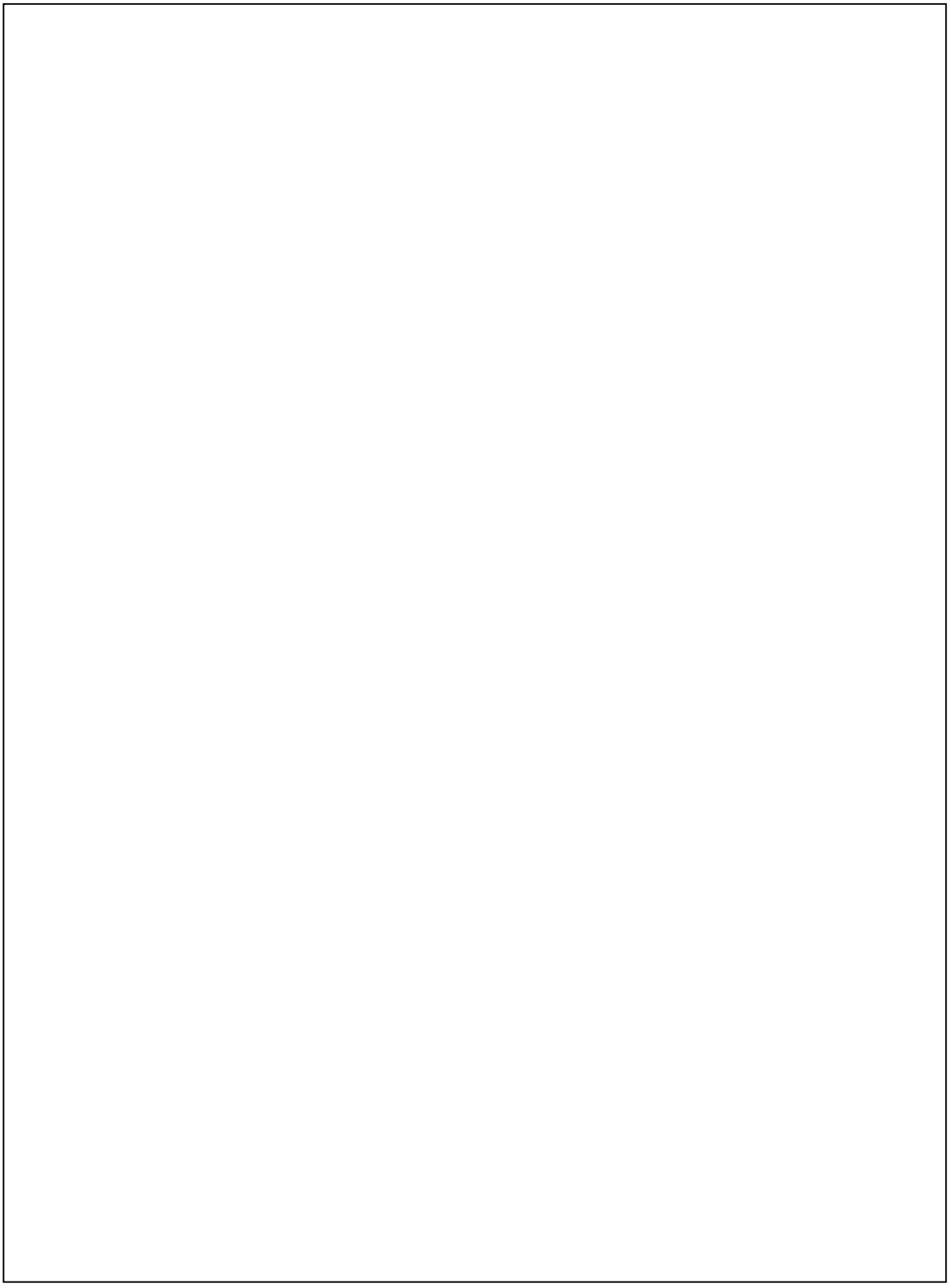
$f(x) = \sin(x^2) + \log_{(\sqrt{x+1})^3}(\sqrt{x+1})^{60}$ at a = $\sqrt{\pi}$

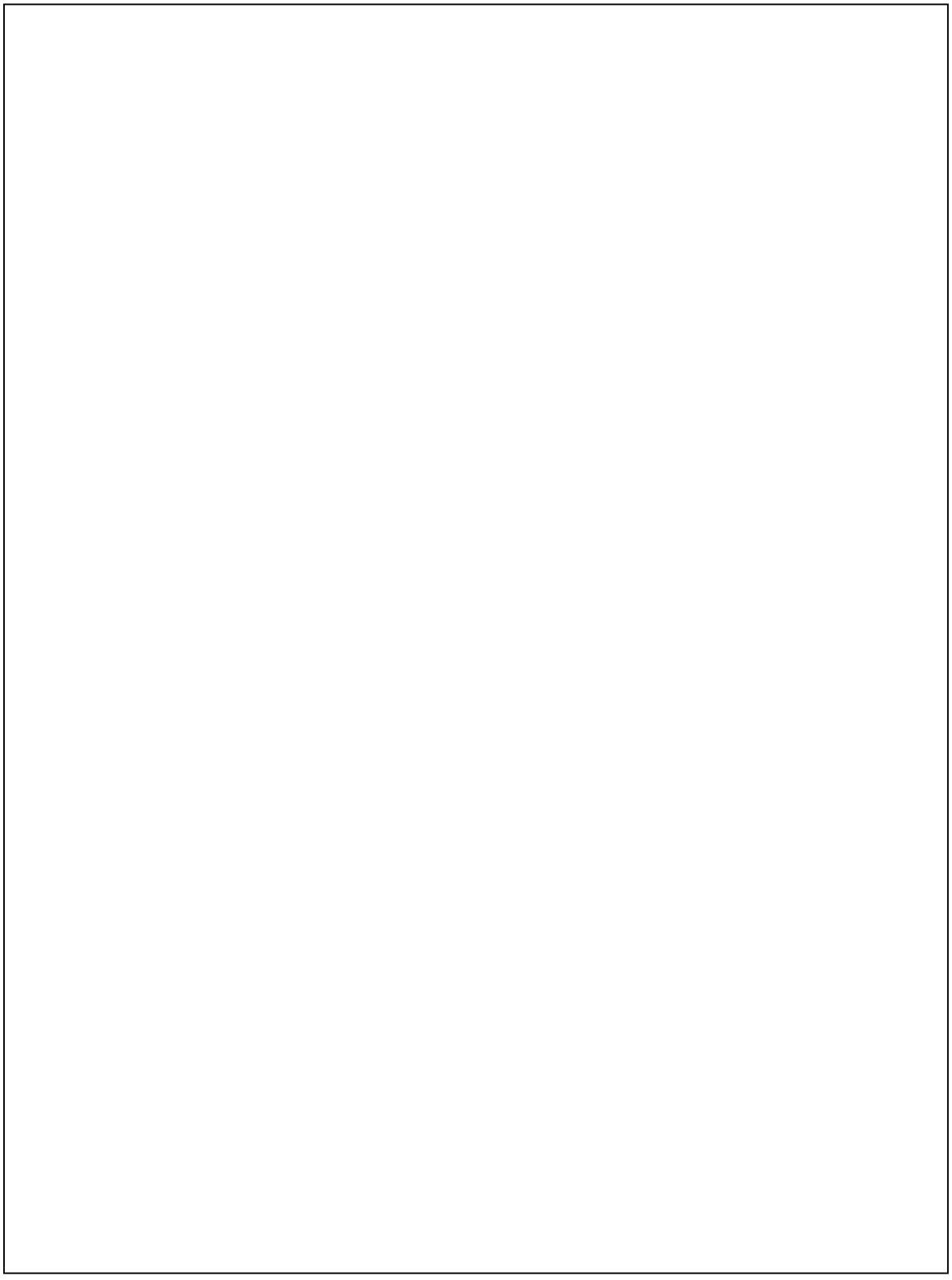
Curve Sketching

Sketch the curve $f(x) = 3x^5 - 5x^3$



Sketch $f(x) = \frac{x}{\sqrt{x^2+1}}$, given $f'(x) = \frac{1}{\sqrt{(x^2+1)^3}}$ and $f''(x) = -\frac{3x}{\sqrt{(x^2+1)^5}}$





Some Fun Limits

Evaluate the limit: $\lim_{x\to 1^-} \ln|x^4-1| - \ln|-1|$

Let f(x) = $\begin{cases} \sin\sqrt[3]{e} & if\ x = e^{\pi} \\ e^{\pi} & if\ x \neq e^{\pi} \end{cases}$, evaluate the limit $\lim_{x\to e^{\pi}} f(f(x))$.

Give your answer in a calculator ready form

Suppose that $\lim_{x\to c} f(x) = z$, where z is a positive integer, which of the following statements are guaranteed to be true? (You may circle multiple options)

a) $f'(c)\ exists$
b) $f(x)\ is\ continuous\ at\ x = c$
c) $f(x) is\ defined\ at\ x = c$
d) $f(c) = z$
e) $a)\ and\ d)$
f) $b)\ and\ c)$
g) $None\ of\ the\ above$