COMM 204

FINAL EXAM REVIEW SESSION

by Brad Gregory

REVIEWING QUEUING

λ = LR avg input rate

ρ = λ/cμ = LR avg Utilization

c = # of servers

μ = LR avg processing rate of a single server

C_a = σ(A)/E(A) = Coefficient of variance (Arrivals)

E(A) = 1/ λ = Avg inter-arrival time

C_s = σ(S)/E(S) = Coefficient of variance (S)

E(S) = 1/μ = Avg processing time for a server = T_s

Prob. Distribution of Arrivals **/** Prob. Distribution of Service **/** # of Servers

$$I_Q \cong \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho} * \frac{C_a^2 + C_S^2}{2} = \frac{\lambda^2}{\mu(\mu-\lambda)} * \frac{C_a^2 + C_S^2}{2}$$

$$\frac{\lambda}{c\mu}$$

Little's Law

$$I_q + I_s = I$$

$$I_q = \lambda \times T_q \updownarrow \qquad I_s = c\mu \times T_s \updownarrow \qquad I = \lambda \times T \updownarrow$$

$$T_q + T_s = T$$

PRACTICE

Matt just finished finals and would like a beverage to celebrate but doesn't want to wait too long. Since he is a frequent customer at this beverage establishment, he has some information about the queue. On average, he estimates that 3 customers arrive every minute. He assumes that these arrivals are exponential. Each server can consistently serve exactly 2 customers per minute, with negligible variance. On this particular day, there are 2 servers at the counter. How many seconds will Matt have to wait in line?

M/D/2 queue

$\rho = \frac{\lambda}{c\mu} = \frac{3}{2\times 2} = 0.75$

$$I_q \cong \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho} * \frac{C_a^2 + C_S^2}{2} = \frac{.75^{\sqrt{2(2+1)}}}{1-.75} * \frac{1+0}{2} = 0.9886$$

$$T_q = \frac{I_Q}{\lambda} = \frac{.9886}{3} = 0.3295 mins \times 60s/\min = \mathbf{19.77\ seconds}$$

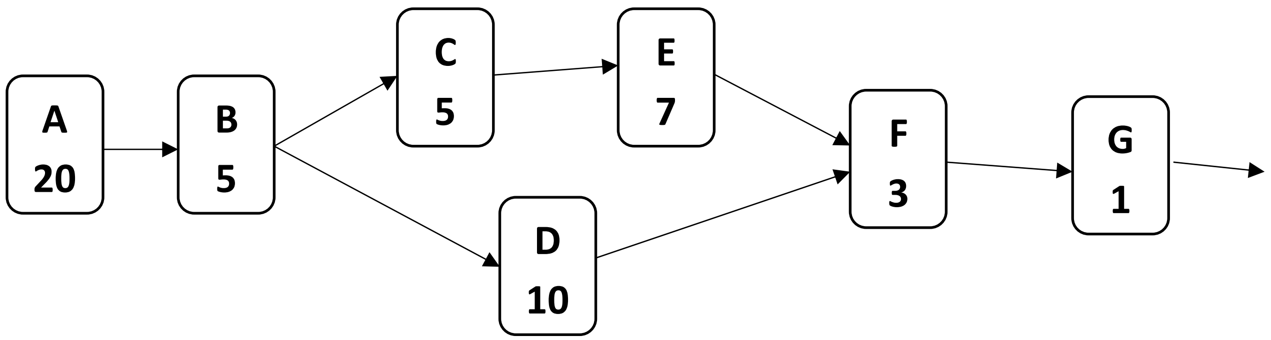
Matt would like to make friends during the time between him lining up and him leaving the system with his beverage. How many potential friends will there be in the **entire process** on average at a single moment in time?

$I_q = 0.9886$

$I_s = \rho = 0.75$

$I = I_q + I_s = 0.9886 + 0.75 = \mathbf{1.7386\ people}$

PROJECT MANAGEMENT



What is the length of the Critical Path? 20 + 5 + **5** + **7** + 3 + 1 = **<u>41</u>**

How much will it cost us to complete the project in 30 days?

Activity	Crash Time (days)	Crash Cost ($)	Crash Cost/Time ($/day)
A	10	$300	$30
B	4	$300	$300
C	Can't be Crashed		
D	4	$60	$10
E	3	$100	$25
F	Can't be Crashed		
G			

Crash Time: Minimum possible time to complete an activity

Crash Cost: Cost associated with the crash time

What do we crash first?

NOT D → not part of our critical path → E!

We crash Activity E by 2 days for a total cost of $50

Our project length is now 39 days.

What next?

We crash Activity A by 9 days for a total cost of $270

Our project length is now 30 days!

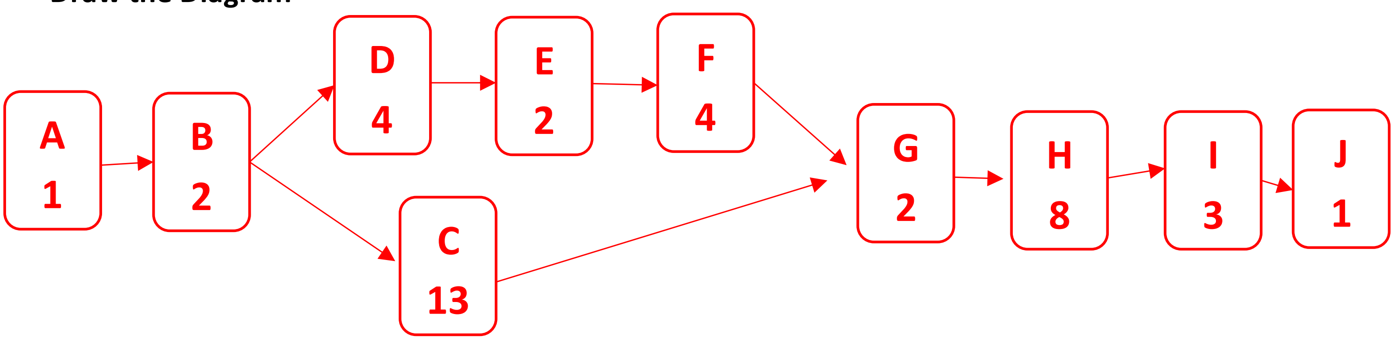
Our total cost is $50 + $270 = $320

PRACTICE

Fredrik has left all of his studying for his COMM 204 exam to 24 hours before it (unlike all of you). He also has other tasks to do to keep his mind healthy and fresh for the exam. How much will he have to pay to fit it all in 24 hours?

Code	Activity	Time (hours)	Immediate Predecessors	Crash Time (hours)	Crash Cost ($)	$/hr
A	Review CMP	1	None	Not Crashable		
B	Breakfast	2	A	1	$10	$10
C	Review Slides	13	B	8	$500	$100
D	Study Date	4	B	Not Crashable		
E	Lunch	2	D	1	$15	$15
F	Laundry	4	E	0	$160	$40
G	Dinner	2	C, F	1	$20	$20
H	Sleep	8	G	Not Crashable		
I	Practice	3	H	2	$150	$150
J	Cheat Sheet	1	I	Not Crashable		

Draw the Diagram



Cost	# hours	Critical Path	Crashable Tasks	Best Option
$0	30	ABCGHIJ	BCGI	B
$10	29	ABCGHIJ	CGI	G
$30	28	ABCGHIJ	CI	C
$330	25	Both	CEFI	C & E
$445	24	-	-	-

FORECASTING

Methods:

Simple Moving Average: $F_t = \frac{A_{t-1}+A_{t-2}+A_{t-3}+\cdots+A_{t-n}}{n}$

- *n* period moving average

Weighted Moving Average: $F_t = w_1 \times A_{t-1} + w_2 \times A_{t-2} + \cdots + w_n \times A_{t-n}$

- Make sure w's all add to 1

Exponential Smoothing: $F_t = F_{t-1} + \alpha \times (A_{t-1} - F_{t-1})$

- α = % of the prior error term to include

How do I know if my forecast is good or not?

Mean Absolute Deviation (MAD): $MAD = \frac{\sum_{t=1}^{n} |F_t - A_t|}{n}$

- Measures the quality of a forecast

where n is the number of prior periods used in the forecast…

Greater n ←——————→ Smaller n

Greater n	Smaller n
Uses more data	Changes more quickly in response to underlying changes
Outliers have less impact	
If **no** trend in the data → results in lower error	If **clear** trend in the data → results in lower error

PRACTICE

Asa is trying to forecast how many pizzas from Mercante's he will eat in December. Help him predict his December consumption using the Weighted Moving Average method. He has collected the following data for the past 10 months:

Month	Pizzas	Weight	
February	1	0	
March	3	0	
April	8	0	
May	5	0	
June	3 x	0.1	= 0.3
July	4 x	0.1	= 0.4
August	8 x	0.1	= 0.8
September	6 x	0.2	= 1.2
October	10 x	0.2	= 2.0
November	12 x	0.3	= 3.6
		1.0	= 8.3

Do you think either assigning different weights to periods or including a different amount of periods would be better? Why or why not?

No "right" answer!

I'd lean towards shifting more weighting towards more recent periods or including less periods since there is a clear upward trend in the data.

Could also argue that October and November are outliers and you should therefore include more periods which would reduce the impact of these.

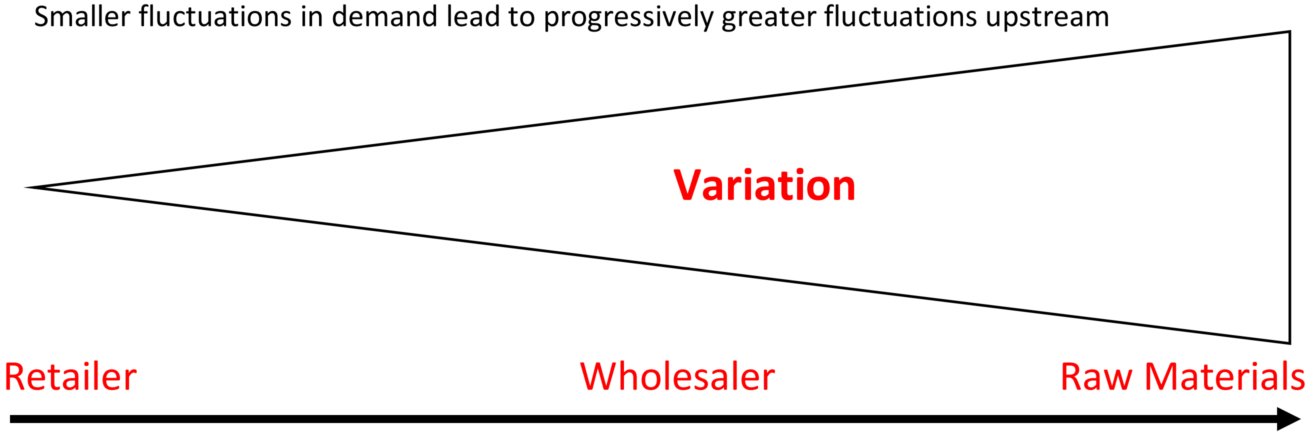
Asa actually consumed a whopping 15 pizzas in December. What was our forecast error?

15 – 8.3 = **6.7**

SUPPLY CHAIN MANAGEMENT

Bullwhip Effect:

Smaller fluctuations in demand lead to progressively greater fluctuations upstream



Why it is bad:

Essentially, it makes planning difficult which is costly:

Inefficient production patterns

Excess inventory

Greater safety stock required

Poor customer service

Hiring/firing of employees

Why it occurs:

Poor ordering practices by retailers:

Order synchronization

Order batching

Trade promotions & forward buying

Reactive ordering

Shortage gaming

How to fix it:

Sharing information: Communication allows upstream suppliers to adjust

Smoothing product flows: Reduce batch sizes, closer to Just-In-Time systems

Eliminating pathological incentives: Make demand itself more consistent

Vendor-Managed Inventory (VMI): Supplier manages retailer's inventory

INVENTORY: ECONOMIC ORDER QUANTITY

INVENTORY	
PROs	**CONs**
Prepare for predictable variability	Holding costs
Insure against unpredictable variability	Ties up working capital
Save $ with bigger batches	Risky to hold large quantities
Inventory in transit	"The Sea of Inventory"
Strategy	

Economic Order Quantity (EOQ):

Tradeoff between **ordering** and **holdings** costs.

How much to order **each time** assuming **constant demand** with **no variability.**

$$EOQ = \sqrt{\frac{2SD}{H}} = \sqrt{\frac{2SD}{iC}}$$

Where:

D = Annual Demand Rate

Q = Lot or batch size

S = Ordering/Set-up Cost

C = Unit cost

H = Holding cost per unit

i = Percent carrying cost (interest rate)

Where do we get the EOQ formula from? Math 104/184! We find the minimum of the Total Cost function and solve for Q.

$$TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

where $\frac{Q}{2}H$ = Holding Cost as a function of order quantity

and $\frac{D}{Q}S$ = Ordering Costs as a function of order quantity

INVENTORY: NEWSVENDOR MODEL

Tradeoff between having **too much** and **too little** inventory.
How much to order **in total** with **varying demand**.

$$\textit{Critical Ratio} = \frac{C_u}{C_u + C_o} = \textit{Prob. of Meeting Demand}$$

Where:

C_u = Cost per unit of demand not met (Opportunity Cost of Lost Sales)

C_o = Cost per unit of excess inventory (Holding Cost)

Stock: $S^* = \mu + \sigma z$

Where:

S^* = Total stock

μ = Mean demand

σ = Standard deviation of demand

z = NORMSINV(**Critical Ratio**) = z-score

PRACTICE

Santa has shifted his business model. He now gives out lumps of coal for bad children as parents order them, as opposed to giving it all out on the same day. There are exactly 500,000 bad kids each year (needing 1 lump of coal each). It costs Santa $1,000 to transport each batch of coal to the North Pole. Santa buys his coal for $20 per lump and sells it to parents for $40 per lump. Santa's elves in the finance department have advised him to use an interest rate of 8%. How much coal should Santa order each time?

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2(1{,}000)(500{,}000)}{(.08)(20)}} = \#\mathbf{25{,}000}$$

Mrs. Clause says that Santa has his ordering cost wrong and that the actual EOQ is #40,000 lumps of coal. What is the correct ordering cost assuming #40,000 is the correct EOQ?

$$EOQ = 40{,}000 = \sqrt{\frac{2(500{,}000)S}{(.08)(20)}}$$

$$40{,}000^2 = \frac{2(500{,}000)S}{(.08)(20)}$$

$$\frac{(.08)(20)40{,}000^2}{2(500{,}000)} = S = \$\mathbf{2{,}560}$$

Randolph the OpLog Reindeer knows that there will be some variation in the number of bad kids. On average, there are 500,000 bad kids per year. However, the standard deviation of bad kids per year is quite large at 75,000. Taking Randolph's points into account, what is the optimal probability of meeting demand?

C_u = $40 - $20 = $20 for the lost sale

C_o = $20i = (20)(.08) = $1.60 for the lost chance to earn interest

$$Critical\ Ratio = \frac{C_u}{C_u+C_o} = \frac{20}{20+1.6} = \mathbf{0.9259\ or\ 92.59\%}$$

Assuming the corresponding z score is 1, how many lumps of coal should Santa stock?

$$S^* = \mu + \sigma z = 500{,}000 + 75{,}000(1) = \#\mathbf{575{,}000}$$

MORE INVENTORY

Average Inventory: $\frac{EOQ}{2}$

Average Flow Time: Little's Law! (I = R x T) → $T = \frac{I}{R} = \frac{\frac{EOQ}{2}}{D}$

Cycle Time (time between orders): $\frac{EOQ}{D}$

Ordering Frequency: $\frac{1}{Cycle\ Time} = \frac{D}{EOQ}$

Lead Time: How long it takes to receive an order once it is placed

Reorder Point: When you should place an order = D x Lead Time

Pipeline Inventory: Same as Reorder Point! = D x LT

- If you pay for it once it *leaves* the supplier, add to your average inventory

Cycle Service Level: Probability that Demand during the Lead Time (DLT) is less than or equal to the Reorder Point. Probability of meeting demand.

Safety Stock: Excess inventory held to increase the probability of meeting demand.

SS = Amount to stock per Newsvendor Model – Mean demand = S^* - E(D)

= Reorder Point – Demand during Lead Time = ROP – D x LT

= z x σ_{LT} where z = NORMSINV(CSL)

PRACTICE

We had the following data from the previous question about Santa and his lumps of coal:

D = #500,000	S = $2,560	P = $40
EOQ = #40,000	C = $20	i = 8%

We also just found out that it takes 20 days from the time Santa places and **pays for** an order until he receives it at the North Pole. Find out:

How often should Santa place an order?

Ordering Frequency = $\frac{D}{EOQ} = \frac{500{,}000}{40{,}000} = \mathbf{12.5\ orders/year}$

How many **days** should Santa wait between placing orders?

Cycle Time = $\frac{1}{Ordering\ Freq.} = \frac{1}{12.5} = 0.08\ yrs \times 365 \frac{days}{year} = \mathbf{29.2\ days}$

Once how many units are left in inventory should Santa place an order?

Reorder Point = $D \times LT = 500{,}000 \times \frac{20}{365} = \#\mathbf{27,398}$

What is Santa's average inventory?

Average Inventory = $\frac{EOQ}{2} + Pipeline\ I = \frac{40{,}000}{2} + 27{,}398 = \#\mathbf{47,398}$

What is Santa's yearly holding cost?

Holding Cost = $Avg\ I \times iC = 47{,}398 \times .08 \times 20 = \$\mathbf{75,836.80}$

If Santa actually places orders when there are #35,000 units left, what is his Safety Stock?

Safety Stock = $ROP - D \times LT = 35{,}000 - 27{,}398 = \#\mathbf{7,602}$