

**COMM 204**

**2019W1 Midterm Review Package**

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# Table of Contents

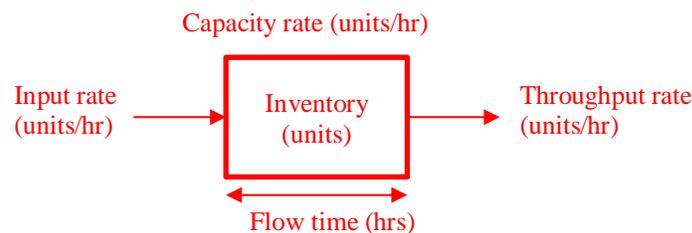
- 1) Process Analysis
- 2) Little's Law
- 3) Utilization
- 4) Inventory
- 5) Product Mix
- 6) Variability in Processes
- 7) OM Triangle
- 8) Queuing



# 1) Process Analysis

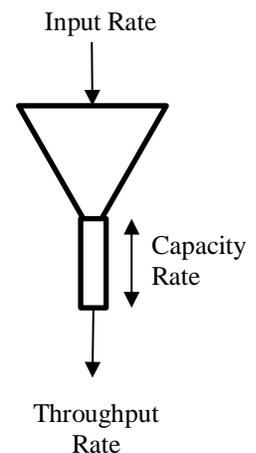
**Process Analysis:** 1) Determine purpose → 2) Process mapping → 3) Capacity Analysis

- **Process flow diagram:** Good tool for understanding & visualization of process
- **Process mapping:** Constructing the process flow diagram
  - > **Flow units:** items flow through the process (ex. Cookies)
  - > **Activities:** transformation steps (ex. Baking)
  - > **Resources & Time:** perform activities (ex. Oven)
  - > **Buffers:** storage flow unit (ex. Counter space)



- **Flow time:** length of time that a unit spends from the beginning to the end for specific “activity” (min, sec, hours...)
- **Bottleneck:** resource w/ lowest capacity rate in a process → Determines the process capacity rate
- **Capacity rate:** maximum possible output rate (units/hr, units/min...)
- **Throughput rate:** actual output rate (flow rate) → Depends on the Capacity rate & Input rate (units/hr, units/min...)

IF D _ Capacity	Throughput Rate
D < Capacity	Smaller of input & capacity
D = Capacity	Throughput = demand rate
D > Capacity	Produce less than D



- **Unit load:** total amount of time the resource works to process (min, sec, hours...)
- **Cycle time:** average time between completion of units (min, sec, hours...)

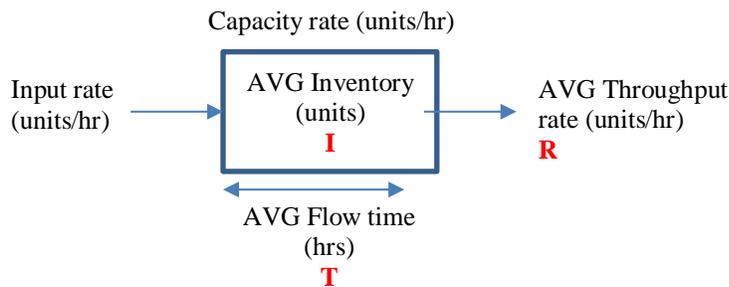
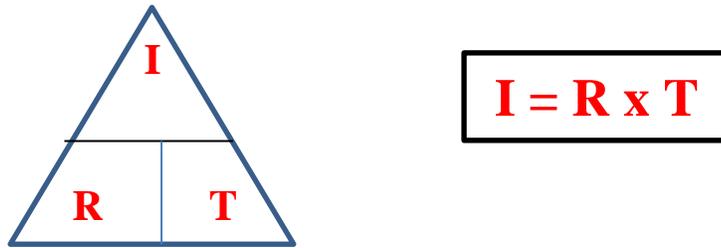
How can we increase the capacity rate of bottleneck?

1. Increase the size of resource pool
2. Decrease the unit load (t)



## 2) Little's Law

- **Little's Law:** relationship between average I, R, T



- A manager can influence any one of them by controlling other two  
→ once two variables are chosen, third is determined
- Reduce flow time (T) by increasing throughput rate (R) and decreasing Inventory (I)

What happens if I decrease the throughput rate (R)?

→ Flow Time (T) Increases OR Inventory (I) Decreases

## 3) Utilization

- **Utilization:** gives us information about “excess capacity”

$$Utilization = \frac{Throughput}{Capacity} = \frac{Actual Output Rate}{Max Output Rate} \rightarrow \text{Always } \leq 100\%$$

Utilization Profile (Resources)

Resource	Capacity Rate	Throughput Rate	Utilization
A	x	y	y/x * 100%

In practice, impossible to have capacity rate = input rate → Utilization < 100%

- **Implied Utilization:** captures excess demand in the short run → allows us to capture the idea of OT

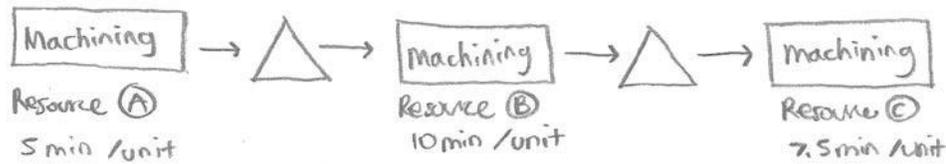
$$Implied Utilization = \frac{Input Rate}{Capacity Rate}$$

SR Avg Input Rate CAN be > SR Avg Capacity Rate



**PRACTICE**

Production process of a manufacturing company



- (a) What is the maximum throughput rate for product?
- (b) What is the cycle time of this process?
- (c) What is the theoretical flow time?
- (d) Assume the operator of Machine C has come up with an innovation that has reduced processing time from 7.5 minutes to 5 minutes.

As a result, the flow time will Decrease, but throughput rate will Stay the Same.

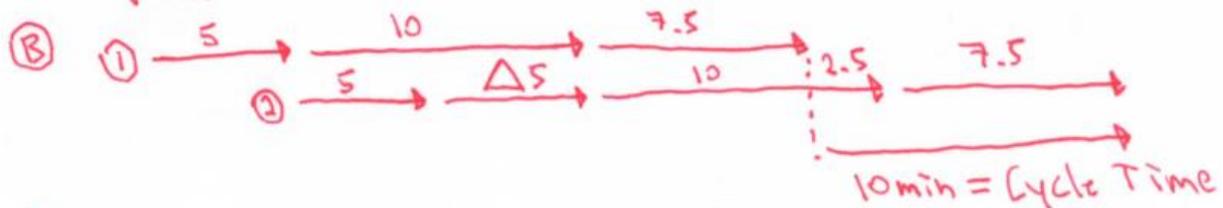
- (e) When the demand rate = capacity rate of the process, what is the short run utilization of resources?
- (f) Assuming input = capacity, what is the average number of units in the process at one time?

**A) WHAT'S THE BOTTLENECK?**

- RESOURCE B = 10 min/unit

$$\frac{10 \text{ min}}{\text{unit}} \times \frac{\text{hr}}{60 \text{ min}} = 0.167 \text{ hr/unit}$$

$$1 / \left( \frac{0.167 \text{ hr}}{\text{unit}} \right) = 6 \text{ units/hr}$$



**C) 5 min + 10 min + 7.5 = 22.5 mins**

**E) UTILIZATION =  $\frac{\text{Throughput}}{\text{Capacity}} = \frac{6}{6} = 100\%$**

WHEN D = CAPACITY, THROUGHPUT = DEMAND

**F) I = R \* T**

R = 6 units/hr

T = 22.5 min \*  $\frac{\text{hr}}{60 \text{ min}}$  = 0.375 hours

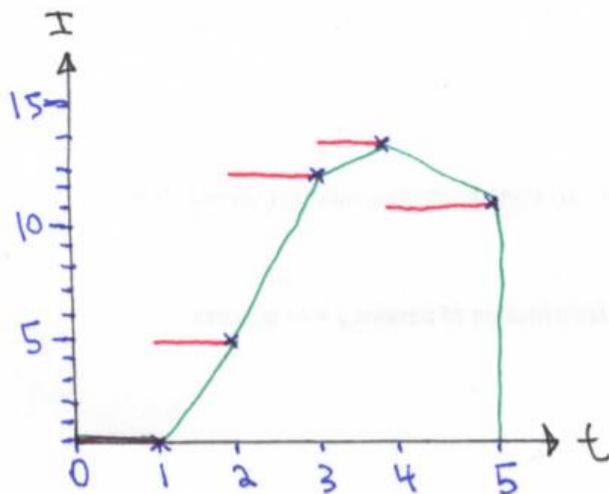
I = 6 \* 0.375 = 2.25



#### 4) Inventory

- Inventory Build up

TIME	INPUT	OUTPUT	CHANGE	INVENTORY
1	3	5	-2	0
2	25	20	+5	5
3	10	2	+8	13
4	8	7	+1	14
5	0	3	-3	11



DISCRETE INVENTORY CHANGE

$$\text{Avg } I = \frac{0 + 5 + 13 + 14 + 11}{5}$$

$$= 8.6$$

CONTINUOUS INVENTORY CHANGE

$$\text{Avg } I = \frac{0 + 5/2 + 9 + 13.5 + 12.5}{5}$$

$$= 7.5$$

- Inventory Turnover

$$\text{Inventory Turnover} = \text{COGS} / I$$

$$\text{Days of Inventory} = 365 * I / \text{COGS}$$



### PRACTICE

Suppose the store opens at 8AM. Customers show up at the rate of 30 per hour until 1PM, and then at the rate of 45 per hour until 3PM. The store closes at 3PM regardless of the number of customers waiting in line, and the unsatisfied customers are sent away. Suppose that every customer who shows up at the store joins the line and waits until satisfied or sent away. The store can serve customers at the rate of 50 per hour between 8AM and 9AM, at the rate of 10 per hour between 9AM and 12 Noon, and then at the rate of 40 per hour between 12 Noon and 3PM. Use the continuous-time model in your calculation.

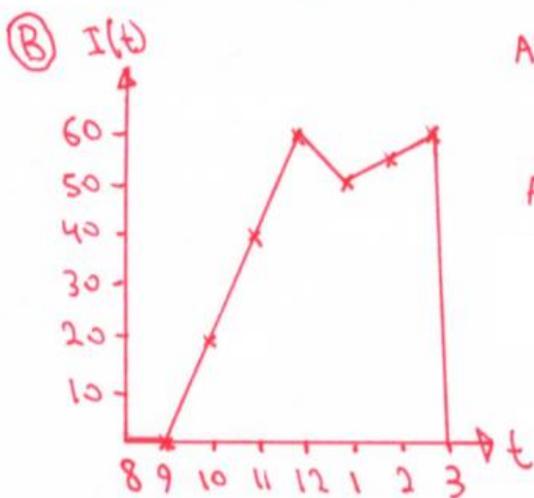
(a) How many customers do you expect to see in the line at 11:30 AM? How many customers are sent away at the end of the day?

(b) Calculate the average number of waiting customers between 8AM and 3PM.

(A)

TIME	INPUT	OUTPUT	CHANGE	INVENTORY
9	30	50	-20	-
10	30	10	+20	20
11	30	10	+20	40
12	30	10	+20	60
1	30	40	-10	50
2	45	40	+5	55
3	45	40	+5	60

AT 11:  $I = 40$ , 12:  $60 \Rightarrow 11:30: I = \frac{60+40}{2} = 50$   
 AT 3:  $I = 60$



$$\text{AREA} = \frac{60 \times 3}{2} + \frac{60+50}{2} + \frac{2(60+50)}{2}$$

$$= 255$$

$$\text{Avg } I = 255 / 7 = 36.43$$



## 5) Product Mix

- When multiple flow units go through a process, **Product Mix** needs to be considered. Bottleneck depends on the Product mix.

Resource	Unit Load (min/unit)			
	A	B	C	Mix (1A + 2B + 2C)
I	1	2	3	$1(1) + 2(2) + 2(3)$ =11
II	0	5	0	$1(0) + 2(5) + 2(0)$ =10
III	4	3	2	$1(4) + 2(3) + 2(2)$ =14 <b>*BOTTLENECK*</b>

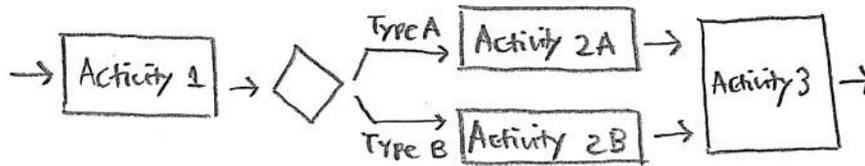
- Flow Shop** Vs. **Job Shop**

Process Type	<u>FLOW SHOP</u>	<u>JOB SHOP</u>
Labour Skills	Low	High
Equipment Specialization	High	Low
Unit Load for each resource	Small	High
Volume	High	Low
Compete on	Cost	Serving Needs
Resources are	Specialized	Flexible
Customization	Standardized	Custom orders
Process Flow Diagram		



### PRACTICE

Consider the following flow diagram for processing a customer order, where each order is either Type A or Type B.



The Exact Processing times and the resources needed for each activity are listed in the table below

Activity	Time (mins)	Used Resources
1	5	Category I
2A	5	Category I
2B	15	Category II
3	5	Category I

There is one unit of resource category I in the system and there are two units of resource category II in the system. Customers currently arrive at the rate of 2.5 per hour; one-fifth of the customers are Type A, and four-fifths of the customers are Type B.

(a) what is the bottleneck resource?

RESOURCE	A	B	PRODUCT MIX
I	15min	10min	$\frac{1}{5}(15) + \frac{4}{5}(10) = 11\text{min}$
II	-	15min	$\frac{1}{5}(0) + \frac{4}{5}(15) = 12\text{min}$
CAPACITY (I) = $\frac{1}{11} = 0.091\text{ orders/min}$ ← Bottleneck = I			
Capacity (II) = $\frac{1}{12} \times 2 = 0.167\text{ orders/min}$			

(b) what are the implied utilizations for each resource listed above?

$$IV = \frac{\text{Input}}{\text{Capacity}} \quad \text{Input} = 2.5/\text{hour} \times \frac{\text{hr}}{60\text{min}} = 0.042/\text{min}$$

$$IV_I = \frac{0.042}{0.091} = 46.2\%$$

$$IV_{II} = \frac{0.042}{0.167} = 25.1\%$$



## 6) Variability in Processes

- Variability comes from input and capacity rates

Predictable Variability	Unpredictable Variability
“Knowable”	“Unknowable”
Can be controlled by making changes to the system	Result of the lack of knowledge or information

- **With no buffer:**

Throughput < Input Rate

- **Effect of Input variability with buffer**

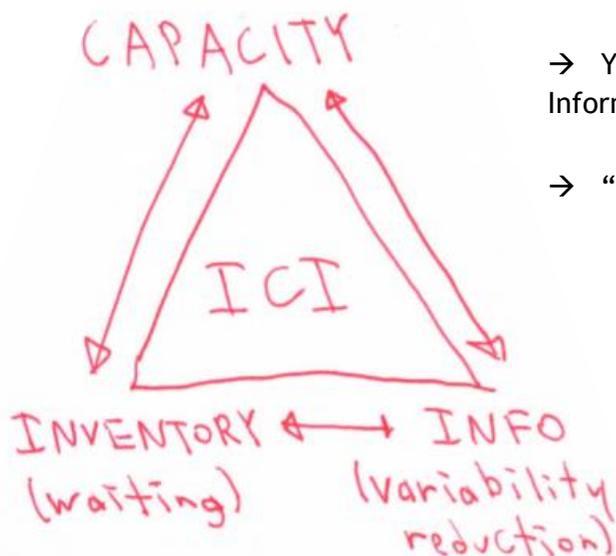
Throughput = Input Rate

## 7) The OM Triangle

→ If a firm is striving to meet the Random D, then we can use CIC as substitutions

→ You cannot have low inventory, capacity, Information acquisition at the same time

→ “TRADE OFF



## 8) Queuing

- Pollaczek – Khinchin (P-K) Formula

$$I_Q \cong \frac{\rho \sqrt{2(c+1)}}{1-\rho} * \frac{C_a^2 + C_s^2}{2} = \frac{\lambda^2}{\mu(\mu-\lambda)} * \frac{C_a^2 + C_s^2}{2}$$

Where:

$\lambda$  = LR avg input rate

$\rho = \lambda/c\mu =$  LR avg Utilization

$c =$  # of servers

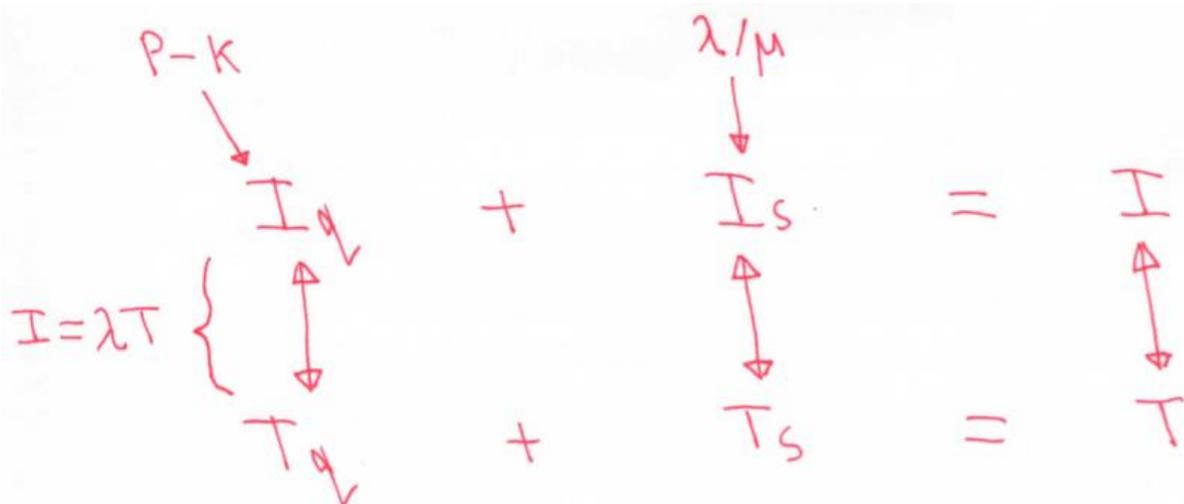
$\mu =$  LR avg processing rate of a single server

$C_a = \sigma(A)/E(A) =$  Coefficient of variance (Arrivals)

$E(A) = 1/\lambda =$  Avg inter-arrival time

$C_s = \sigma(S)/E(S) =$  Coefficient of variance (S)

$E(S) = 1/\mu =$  Avg processing time for a server =  $T_s$



## Queuing Theories

### PK Formula Assumptions

- Single Server / queue / no limit on queuing length - Arrived people stay until get served. FIFO

Form:

Probability Distribution of Arrivals / Probability Distribution of Service / # of Servers

Coefficients of Variance ( $C_a$  or  $C_s$ ):

G  $\rightarrow$  Have to solve

M  $\rightarrow$  CV = 1

D  $\rightarrow$  CV = 0

1. G / G / 1 Queue

$$I_q \approx \frac{\rho^2}{1-\rho} \times \frac{C_a^2 + C_s^2}{2}$$

2. M / M / 1 Queue

$$I_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

3. M / D / 1 Queue

$$I_q = \frac{\rho^2}{1-\rho} \times \frac{1}{2} = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

4. G / G / C Queue

$$I_q \approx \frac{\rho^{\sqrt{2(C+1)}}}{1-\rho} \times \frac{C_a^2 + C_s^2}{2}$$

5. M / M / 3 Queue

$$I_q = \frac{\rho^{\sqrt{2(3+1)}}}{1-\rho}$$

6. D / D / 2 Queue

$$I_q \approx \frac{\rho^{\sqrt{2(2+1)}}}{1-\rho} \times \frac{0}{2} = 0$$

### Dealing with Variability

- Reduce it by improving information about input & Process (D & S)
- Manage it by choosing an appropriate buffer
- Adopt risk pooling  $\rightarrow$  decreases queue length dramatically



## PRACTICE

The bank is consider opening a drive-through window for customer service. Management estimates that customers will arrive at the rate of 15/hour. The teller who will be staffing the window can serve customers at the rate of one every three minutes. Assuming Poisson arrivals and exponential service, find:

(a) Utilization of the teller

(b) Average number in the waiting line

(c) Average number in the system

(d) Average waiting time in line

(e) Average waiting time in the system including service

(f) Suppose now that the service times are deterministic (at the same rate). Which of the above answers would change, and what would be the new values for these answers?

$$M/M/1$$

$$INPUT = \lambda = 15/h$$

$$SERVICE = \frac{1}{\mu} = 3 \text{ min/customer}$$

$$\frac{3 \text{ min}}{\text{customer}} \times \frac{\text{hr}}{60 \text{ min}} = 0.05 \text{ h/customer}$$

$$1/0.05 = 20 \text{ customers/hour}$$

$$\textcircled{A} \rho = \frac{\lambda}{\mu} = \frac{15}{20} = 75\%$$

$$\textcircled{B} I_q = \frac{\rho^2}{1-\rho} = \frac{0.75^2}{1-0.75} = 2.25$$

$$\textcircled{C} I = I_q + I_s = 2.25 + 0.75 = 3$$

$$\textcircled{D} T_q = I_q / \lambda = 2.25 / 15 = .15 \text{ h or } 9 \text{ min}$$

$$\textcircled{E} T = T_q + T_s = .15 \text{ h} + .05 = 0.2 \text{ h or } 12 \text{ min}$$

$$\textcircled{F} M/D/1 \quad I_q = \frac{\rho^2}{1-\rho} \times \left(\frac{1}{2}\right) \Rightarrow I_q \text{ REDUCED BY HALF}$$

$$\textcircled{A} \text{ No CHANGE}$$

$$\textcircled{B} 2.25/2 = 1.125$$

$$\textcircled{C} I = 1.125 + .75 = 1.875$$

$$\textcircled{D} T_q: 9/2 = 4.5 \text{ min}$$

$$\textcircled{E} T = 12 - 4.5 = 7.5 \text{ min}$$



### **BONUS! Tips & Tricks:**

#### **1) Think about units**

Consider what units your answer needs to be in. Set up a formula that will give you an answer in those units.

#### **2) Try not to use your formula sheet**

Make a really nice formula sheet that works for you, then avoid using it. Making a cheat sheet is a great way to identify key ideas. However, during a time-limited exam, it takes a long time to check it when answering each question.

#### **3) Review → Practice, Review → Practice, Review → Practice**

Practice each concept after you review it. Science says that reviewing and then practicing is more effective than reviewing twice. Try to practice in exam-like scenarios

#### **4) Take care of yourself!**

Get a good sleep, go for a walk before the exam, hang out with people. Chances are you'll have to problem solve in the exam, so you want your brain at it's best. That extra hour of studying at 3am the night before is probably not going to help you.

