



commerce
undergraduate
society



COMM 204

MIDTERM REVIEW SESSION

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PROCESS ANALYSIS

Process flow diagram

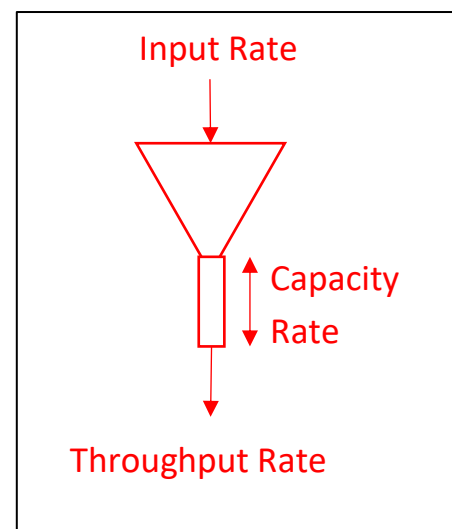
- Flow units: items flowing through the process (ex. Cookies) →
- Activities: transformation steps (ex. Baking) □
- Resources: perform activities (ex. Oven)
- Buffers: storage flow unit (ex. Counter space) △
- Decision points: split-off (ex. Chocolate chips?) ◇

Capacity analysis

- Flow time: the time a unit spends in the whole process (ex. 30 min)
- Unit load: the time a unit spends in one activity (ex. 10 min)
- Capacity rate: Maximum possible output rate (ex. 20 cookies/hour)
- Bottleneck: Resource with lowest capacity rate
 - Determines the process capacity rate
- Input rate: Rate of arrival into the system (ex. 30 cookies/hour)
- Throughput rate: Actual output rate (ex. 20 cookies/hour)
 - Equal to the lesser of Input rate and Capacity rate
 - Can't produce more than capacity, and won't produce more than demand
- Cycle time: average time between completion of units (ex. 10 min)

How can we increase the throughput rate?

- Raise the input rate
 - (ex. Marketing campaign for cookies)
- Decrease the bottleneck's unit load
 - (ex. Superfast oven)
- Add another bottleneck resource
 - (ex. Buy another oven)



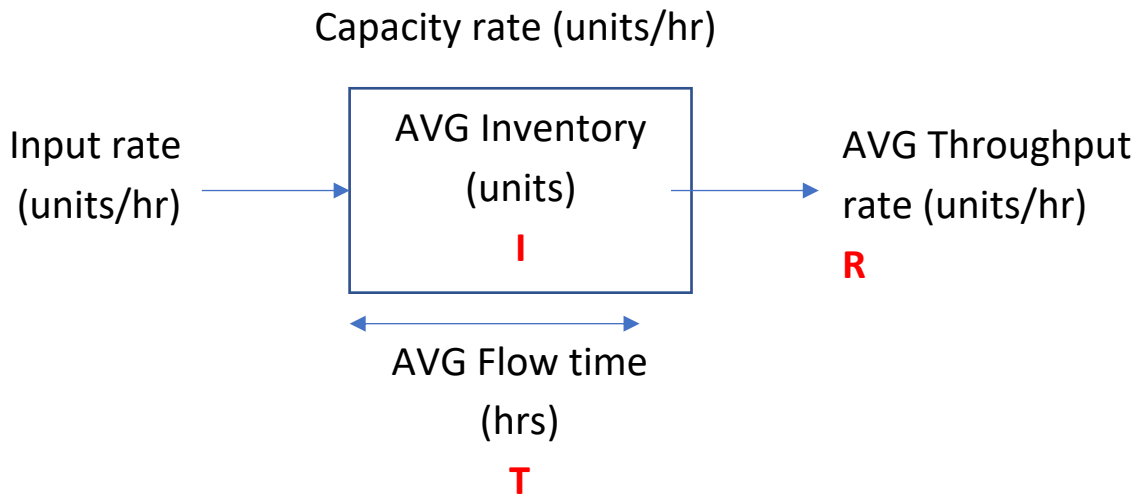
PRACTICE

Draw a Process Flow Diagram of how to make your favourite food and explain it to someone next to you! (if you can't cook, guess)



LITTLE'S LAW

$$I = R \times T$$



UTILIZATION

Tells us about excess capacity:

$$\text{Utilization} = \frac{\text{Throughput}}{\text{Capacity}} = \frac{\text{Actual Output Rate}}{\text{Max Output Rate}} \leq 100\%$$

- Utilization = 100% is impossible in practice \rightarrow Utilization < 100%

Implied Utilization:

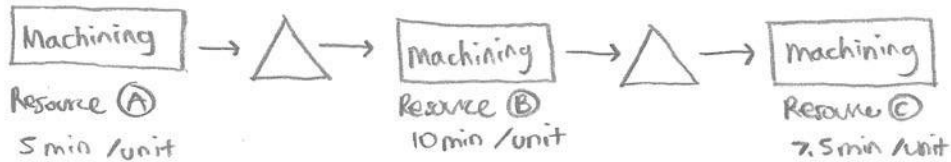
$$\text{Implied Utilization} = \frac{\text{Input Rate}}{\text{Capacity Rate}}$$

- Capture excess demand in the **short run**, but not sustainable in the long run



PRACTICE

Here is the production process of a manufacturing company:



a) What is the maximum throughput rate?

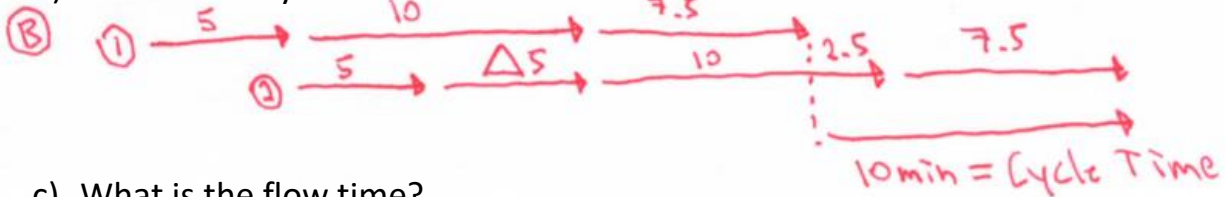
Ⓐ WHAT'S THE BOTTLENECK?

- RESOURCE B = 10 min/unit

$$\frac{10 \text{ min}}{\text{unit}} \times \frac{\text{hr}}{60 \text{ min}} = 0.167 \text{ hr/unit}$$

$$1 / \left(\frac{0.167 \text{ hr}}{\text{unit}} \right) = 6 \text{ units/hr}$$

b) What is the cycle time?



c) What is the flow time?

Ⓒ $5 \text{ min} + 10 \text{ min} + 7.5 = 22.5 \text{ mins}$

d) Assume the operator of Machine C has come up with an innovation that has reduced processing time from 7.5 to 5 min. What will happen to flow time? Throughput rate?

Flow time: Decrease

Throughput rate: Stays the same



PRACTICE CONTINUED

- e) If the input rate = the capacity rate, what is the short run utilization of resources?

$$\textcircled{E} \text{ UTILIZATION} = \frac{\text{Throughput}}{\text{Capacity}} = \frac{6}{6} = 100\%$$

WHEN $D = \text{CAPACITY}$, $\text{THROUGHPUT} = \text{DEMAND}$

- f) If the input rate = the capacity rate, what is the average number of units in the process at one time?

$$\textcircled{F} I = R \times T$$

$$R = 6 \text{ units/hr}$$

$$T = 22.5 \text{ min} \times \frac{\text{hr}}{60 \text{ min}} = 0.375 \text{ hours}$$

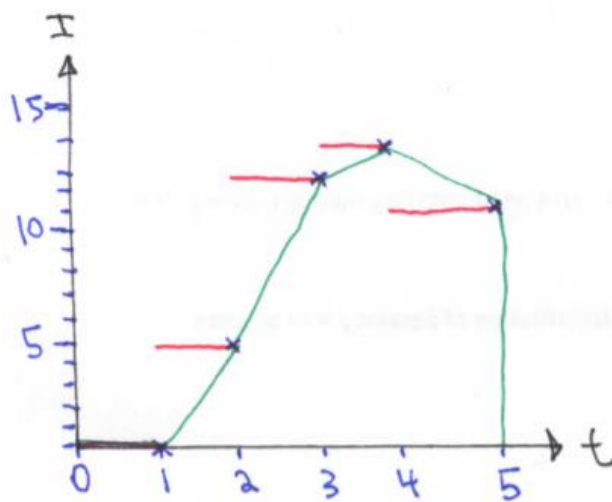
$$I = 6 \times 0.375 = 2.25$$



INVENTORY

Inventory Build-Up

Time	Input	Output	Change	Inventory
1	3	5	-2	0
2	25	20	+5	5
3	10	2	+8	13
4	8	7	+1	14
5	0	3	-3	11



DISCRETE INVENTORY CHANGE

$$\text{Avg } I = \frac{0 + 5 + 13 + 14 + 11}{5}$$

$$= 8.6$$

CONTINUOUS INVENTORY CHANGE

$$\text{Avg } I = \frac{0 + 5/2 + 9 + 13.5 + 12.5}{5}$$

$$= 7.5$$

Inventory Turnover

$$\text{Inventory Turnover} = \text{COGS} / I$$

$$\text{Days of Inventory} = 365 * I / \text{COGS}$$



PRACTICE

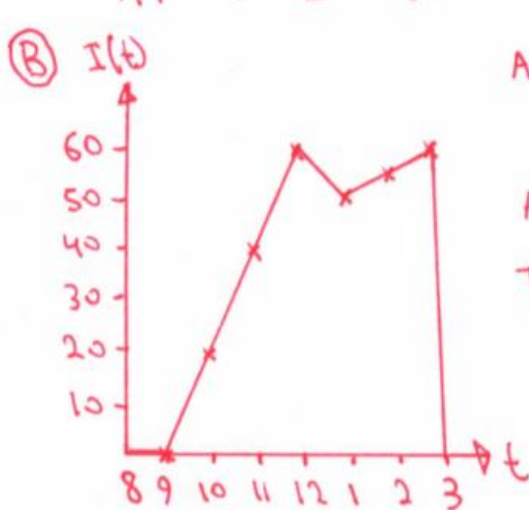
Suppose the store opens at 8AM. Customers show up at the rate of 30 per hour until 1PM, and then at the rate of 45 per hour until 3PM. The store closes at 3PM regardless of the number of customers waiting in line, and the unsatisfied customers are sent away. Suppose that every customer who shows up at the store joins the line and waits until satisfied or sent away. The store can serve customers at the rate of 50 per hour between 8AM and 9AM, at the rate of 10 per hour between 9AM and 12 Noon, and then at the rate of 40 per hour between 12 Noon and 3PM. Use the continuous-time model in your calculation.

- How many customers do you expect to see in the line at 11:30 AM? How many customers are sent away at the end of the day?
- Calculate the average number of waiting customers between 8AM and 3PM.

(A)

TIME	INPUT	OUTPUT	CHANGE	INVENTORY
9	30	50	-20	-
10	30	10	+20	20
11	30	10	+20	40
12	30	10	+20	60
1	30	40	-10	50
2	45	40	+5	55
3	45	40	+5	60

AT 11: $I = 40$, 12: $60 \Rightarrow 11:30: I = \frac{60+40}{2} = 50$
 AT 3: $I = 60$



$$\text{AREA} = \frac{60 \times 3}{2} + \frac{60+50}{2} + \frac{2(60+50)}{2}$$

$$= 255$$

$$\text{Avg } I = 255 / 7 = 36.43$$

THROUGHPUT:
 $30 \times 5 + 4$



PRODUCT MIX

Consider any time there are multiple different flow units

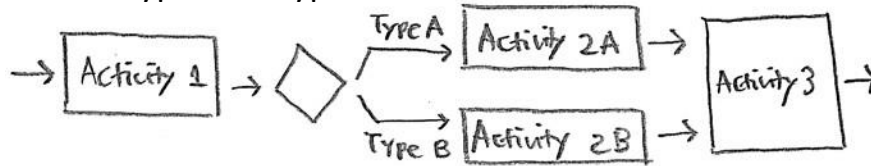
Resource	Unit Load (min/unit)			Mix (1A + 2B + 2C)
	A	B	C	
I	1	2	3	$1(1) + 2(2) + 2(3) = 11$
II	0	5	0	$1(0) + 2(5) + 2(0) = 10$
III	4	3	2	$1(4) + 2(3) + 2(2) = 14$ *BOTTLENECK*

Process Type	<u>FLOW SHOP</u>	<u>JOB SHOP</u>
Labour Skills	Low	High
Equipment specialization	High	Low
Unit Load for each resource	Small	High
Volume	High	Low
Compete on	Cost	Serving Needs
Resources are	Specialized	Flexible
Customization	Standardized	Custom orders
Process flow diagram	<pre> graph TD Start(()) --> 1[1 WA] 1 --> 2[2 WB] 2 --> 3[3 WC] 3 --> End(()) </pre>	<pre> graph TD Start(()) --> 1[1] 1 --> 2[2] 2 --> 3[3] 3 --> End(()) subgraph WA 1 2 3 end </pre>



PRACTICE

Consider the following flow diagram for processing a customer order, where each order is either Type A or Type B:



The exact processing times and the resources needed for each activity are listed in the table below:

Activity	Time (mins)	Used Resources
1	5	I
2A	5	I
2B	15	II
3	5	I

There is one unit of resource I in the system and there are two units of resource II in the system. Customers currently arrive at the rate of 2.5 per hour; one-fifth of the customers are Type A, and four-fifths of the customers are Type B.

a) What is the bottleneck resource?

RESOURCE	A	B	PRODUCT MIX
I	15min	10min	$\frac{1}{5}(15) + \frac{4}{5}(10) = 11\text{min}$
II	-	15min	$\frac{1}{5}(0) + \frac{4}{5}(15) = 12\text{min}$

$\text{Capacity (I)} = \frac{1}{11} = 0.091 \text{ orders/min} \leftarrow \text{Bottleneck} = \text{I}$
 $\text{Capacity (II)} = \frac{1}{12} \times 2 = 0.167 \text{ orders/min}$

b) What are the implied utilizations for each resources listed above?

$$IV = \frac{\text{Input}}{\text{Capacity}} \quad \text{Input} = 2.5/\text{hour} \times \frac{\text{hr}}{60\text{min}} = 0.042/\text{min}$$

$$IV_I = \frac{0.042}{0.091} = 46.2\%$$

$$IV_{II} = \frac{0.042}{0.167} = 25.1\%$$



VARIABILITY IN PROCESS

Predictable Variability	Unpredictable Variability
"Knowable"	"Unknowable"
Can be controlled by making changes to the system	Result of the lack of knowledge or information

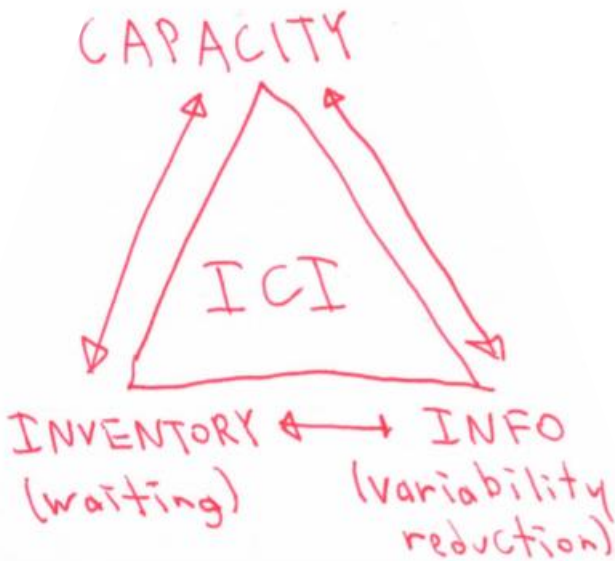
With no buffer:

Throughput < Input rate

With buffer:

Throughput = Input rate

OM TRIANGLE



Trade off between inventory, capacity, and information to meet a variable input rate



QUEUING

P-K Formula

$$I_Q \cong \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho} * \frac{C_a^2 + C_s^2}{2} = \frac{\lambda^2}{\mu(\mu-\lambda)} * \frac{C_a^2 + C_s^2}{2}$$

Where:

λ = LR avg input rate

ρ = $\lambda/c\mu$ = LR avg Utilization

c = # of servers

μ = LR avg processing rate of a single server

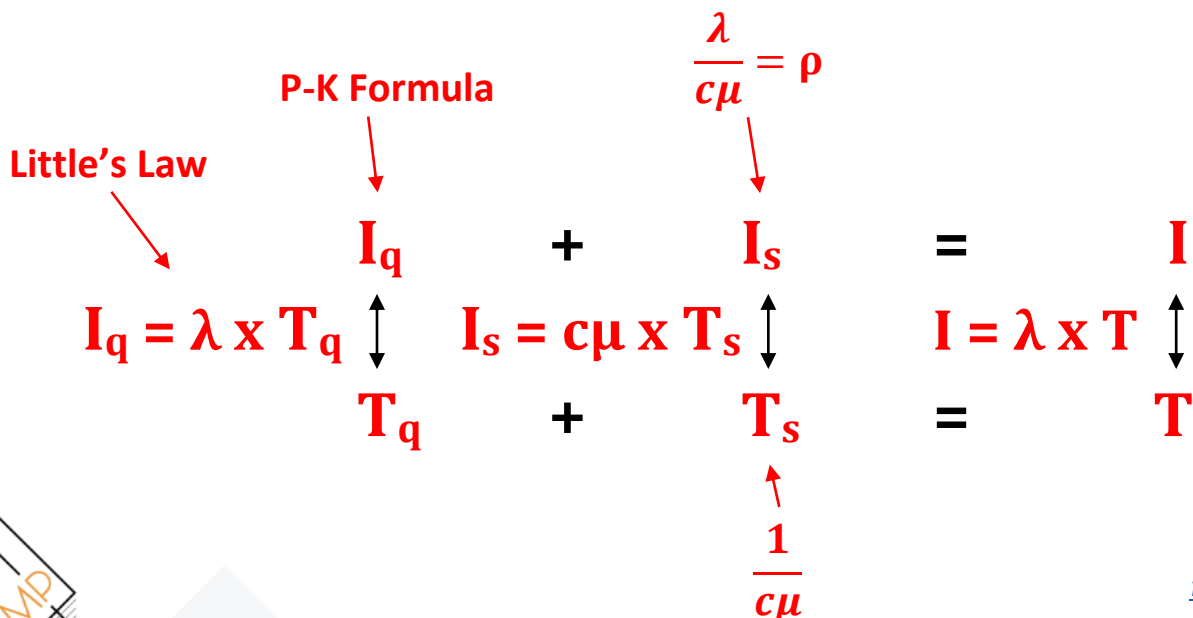
C_a = $\sigma(A)/E(A)$ = Coefficient of variance (Arrivals)

$E(A)$ = $1/\lambda$ = Avg inter-arrival time

C_s = $\sigma(S)/E(S)$ = Coefficient of variance (S)

$E(S)$ = $1/c\mu$ = Avg processing time for a server = T_s

How to find ANY variable in queuing:



QUEUING THEORIES

Prob. Distribution of Arrivals / Prob. Distribution of Service / # of Servers

Coefficients of variance (C_a or C_s):

D \rightarrow 0

M \rightarrow 1

G \rightarrow Have to solve for it

G / G / 1

$$I_q \approx \frac{\rho^2}{1-\rho} \times \frac{C_a^2 + C_s^2}{2}$$

M / M / 1

$$I_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

M / D / 1

$$I_q = \frac{\rho^2}{1-\rho} \times \frac{1}{2} = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

G / G / C

$$I_q \approx \frac{\rho^{\sqrt{2(C+1)}}}{1-\rho} \times \frac{C_a^2 + C_s^2}{2}$$

M / M / 3

$$I_q = \frac{\rho^{\sqrt{2(3+1)}}}{1-\rho}$$

D / D / 2

$$I_q \approx \frac{\rho^{\sqrt{2(2+1)}}}{1-\rho} \times \frac{0}{2} = 0$$

How to reduce variability:

Gather more information about input & capacity

Manage it with a buffer

Risk pooling \rightarrow decreases queue length dramatically



PRACTICE

The bank is consider opening a drive-through window for customer service. Management estimates that customers will arrive at the rate of 15/hour. The teller who will be staffing the window can serve customers at the rate of one every three minutes. Assuming Poisson arrivals and exponential service, find:

- a) Utilization of the teller
- b) Average number waiting in line
- c) Average number in the system
- d) Average waiting time in line
- e) Average waiting time in the entire system

M M

M/M/1

INPUT = $\lambda = 15/h$

SERVICE = $\frac{1}{\mu} = 3 \text{ min/customer}$

$\frac{3 \text{ min}}{\text{customer}} \times \frac{\text{hr}}{60 \text{ min}} = 0.05 \text{ h/customer}$

$\frac{1}{0.05} = 20 \text{ customers/hour}$

Ⓐ $\rho = \frac{\lambda}{\mu} = \frac{15}{20} = 75\%$

Ⓑ $I_q = \frac{\rho^2}{1-\rho} = \frac{0.75^2}{1-0.75} = 2.25$

Ⓒ $I = I_q + I_s = 2.25 + 0.75 = 3$

Ⓓ $T_q = I_q / \lambda = 2.25 / 15 = .15 \text{ h or } 9 \text{ min}$

Ⓔ $T = T_q + T_s = .15 \text{ h} + .05 = 0.2 \text{ h or } 12 \text{ min}$



PRACTICE CONTINUED

- f) Suppose now that service times are constant. Which of the previous page's answers would change and what would be their new values?

Ⓕ M/D/1 $I_q = \frac{\rho^2}{1-\rho} \times \left(\frac{1}{2}\right) \Rightarrow I_q$ REDUCED BY HALF

Ⓐ No CHANGE

Ⓑ $2.25/2 = 1.125$

Ⓒ $I = 1.125 + .75 = 1.875$

Ⓓ $T_q: 9/2 = 4.5 \text{ min}$

Ⓔ $T = 12 - 4.5 = 7.5 \text{ min}$



TIPS & TRICKS

1) Think about the units

- Determine what units your answer needs to be in and choose a formula that will give you those units.

2) Try not to use your cheat sheet

- Making a cheat sheet is a great way to isolate the big ideas and have just in case, but it takes time to rely on in an exam

3) Review → Practice → Repeat

- Science says that practicing a concept after reviewing it is the best way to study, bonus points for practicing in an exam-like scenario

4) Take care of yourself

- Your health is more important than your grade! And you'll do better after a good night's sleep, healthy meal, and some time to unwind

Good luck!

