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COMM 298 - Intro to Finance

FINAL REVIEW SESSION

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Overview

- I.** Introduction
- II.** Summaries of the 8 lectures
- III.** Formula Sheet
- IV.** Sample Questions
- V.** Q&A
- VI.** Feedback Form



Lecture Summaries

Intro to Finance

Why do people borrow? Why do people invest? – lack of capital or lack of projects.

Debt ratings and covenants, seniority, callable debt, convertible debt

Common shares, dual shares, preferred shares, voting rights → Absolute Priority Rule of financing

Financial Management Decisions:

Capital budgeting

What long-term investments or projects should the business take on?

Capital structure

How should we pay for our assets? Should we use debt or equity?

Working capital management

How do we manage the day-to-day finances of the firm?

Time Value of Money

Simple vs. compounded interest

Future versus Present Value of an investment

“r” as a return, discount rate, cost of capital etc.

Effective Annual Rate, APR [percent, periods]

Loans (payday, interest-only, pure discount, equal payments, equal principal, amortization table)

Converting from and to EAR, APR, Effective period rates

Annuities, Present and Future Value of them (Annuities Due)

Contract interest rate versus **Personal interest rate**

Bonds

YTM = the **one** discount rate to price bond, quoted in APR terms

YTM assumes: hold the bond until maturity, reinvest coupons at YTM.

YTM as “solver” of finding the one “r” to price bond

Current Yield = Annual Coupon / Current Price

Bond prices quoted as percent of a \$1,000 face value.

Quotes provide coupon rate, maturity date, and YTM

YTM < CR	P > F	a premium
YTM = CR	P = F	par
YTM > CR	P < F	a discount

Default (or credit) risk: risk that a bond issuer will be unable to make promised payments of coupons or principal. Investors expect to be compensated through a higher yield. (Risk **averse**)

Default premium: YTM on risky bond - YTM of risk-free bond w/ similar maturity and coupon.

Realized rate of return (ROR): the annual rate of return that an investor actually earned during the time period that they owned an asset. Find HPR, and then find EAR of it to end with ROR

Bond yields affected by: The real rate of interest, Premium for expected future inflation, Interest rate risk premium, Default risk premium, & Liquidity premium

The **term structure** is a plot of yields on risk-free gov.t bonds of different maturities. (yield curve)

Risk-free bond yields represent: The real rate of interest, Premium for expected future inflation, & An interest rate risk premium

Nominal returns: returns in dollar terms, **Real returns:** returns in terms of purchasing power.

- Investors should be concerned about the real investment rate of return, but **Money Illusion** exists

Inflation = % increase in price of goods, Investors **should** care about purchasing power not price
Discounting nominal cash flows by nominal % = discounting real cash flows by real interest rate!

Stocks

Residual cash flows of a firm = **earnings**, board and management decide how much of earnings to distribute to shareholders. Dividends are the portion of earnings paid to shareholders. Shareholders elect a board of directors (1 share = 1 vote), whose job is to hire management. Preferred shares pay a fixed periodic dividend (perpetuity) and have priority over common shares.

Common Stock price = present value of all expected future dividends $\rightarrow D1/(1+r) + D2/(1+r)^2 \dots$
If zero growth in dividends, then just perpetuity. If Dividends grow, $P(0) = [D1 / (r - g)]$
Growth model assumes $r > g$ (usually true)

Expected rate of return, r , is a market rate, driven by alternative returns on similar-risk securities and is NOT determined by the expected dividends and growth of the firm.

The price of the stock, P_0 , varies and is related to expected dividends and growth prospects, given the market rate of return, r .

Cum-dividend stock price = ex dividend + not issued yet dividend

What if un-even growth? (super high then normal) \rightarrow forecast weird years, find terminal value \rightarrow PV

Price to earnings ratio: ratio of price per share to earnings per share $\rightarrow P/E = \text{Price} / \text{EPS}$

Where $\text{EPS} = \text{Earnings per share} = \text{Net Income} / \# \text{ of shares outstanding}$

(**if given market cap and price you know # of shares)

Simple measure of how investors view a firm's potential growth.

- Low P/E firms are thought to have low future growth.
- High P/E firms are expected to have high future growth

Compare stock's P/E to median or mean multiple for comparables and imply under/overvalued

Realized returns vs. **Expected** returns

Market Efficiency

Efficient Markets Hypothesis: investors use **all available** and relevant information to price assets
(A market is *efficient* if prices quickly and accurately reflect new information)

- **Weak form efficient** if prices reflect all information about past prices
 - Why is technical analysis pointless in a weak form efficient market?
- **Semi-strong form efficient** if prices reflect all publicly available information.
 - Why is fundamental analysis pointless in a semi-strong form efficient market?
- **Strong form efficient** if prices reflect all information, public or private.
 - In strong form efficient market, why no advantage to trading on private information?

Insider Trading: Using *private information* to trade and profit (material, non-public, price sensitive)

Most evidence supports the idea that financial markets are actually **weak-form efficient**

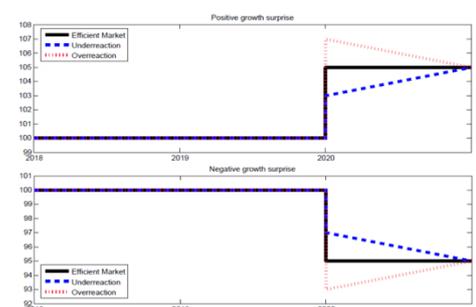
Well-established, liquid markets for well understood securities tend to be semi-strong form efficient. Pockets of inefficiency may exist in newly established, illiquid markets for new misunderstood securities

Expected price based on expected growth rates from probabilities?
Under-reaction vs. over-reaction
(did it under/over shoot the correction? Can be positive or negative)

Sometimes market will over-react (under-react), this is still efficient if on **average** it is correctly priced in.

If consistent under/over reaction, how can we arbitrage?

- If investors over-react to good news, sell \rightarrow wait for adjustment \rightarrow buy-back
- If investors over-react to bad news, buy \rightarrow wait for adjustment \rightarrow sell



Capital Budgeting

In trying to create an investment decision rule:

- Should adjust for time value of money
- Should adjust for risk
- Measure the value created

Payback period = amount of time required for investment to generate cash flows to recover initial cost

Payback rule = if payback period is < than some chosen value, accept project

- Doesn't adjust for risk or time value

Net Present Value (NPV) = the value added today by taking on an investment project (sum of PV's)

If NPV is > 0, accept project because it creates value

- NPV may be difficult to apply, unknown appropriate discount rate, or poor cash flow estimates

Internal Rate of Return (IRR): the discount rate that makes the NPV of an investment equal to zero

IRR rule: accept an investment project if the IRR exceeds the required return.

If $IRR > \text{cost of capital} \rightarrow NPV > 0 \rightarrow \text{accept}$

If $IRR < \text{cost of capital} \rightarrow NPV < 0 \rightarrow \text{decline}$

So, same results as NPV, **ASSUMING** cash flows follow negative to positive pattern

Also, IRR may lead to incorrect decisions when comparing mutually exclusive projects are made

Profitability Index (PI) = PV of investment's future cash flows divided by initial cost (benefit/cost ratio).

PI measures the value created per dollar invested.

Useful if capital is scarce \rightarrow allocate to highest PI projects first

Advantages/Disadvantages of each method? (IMPORTANT)

4 Steps for Capital Budgeting:

1. Forecast a project's future cash flows
2. Obtain an estimate of the project's cost of capital
3. Use the project's cost of capital to discount future cash flows
4. If the NPV of the project is greater than zero, go ahead with the investment

6 Types of Investment projects

1. New products
2. Expansion of existing products
3. Replacement of equipment or building
4. Research and development
5. Exploration
6. Safety-related or pollution-control devices

Always discount **cash flows**, not profits, with NPV always discount when they occur

Profits depend on depreciation and timing of revenue/expense recognition

The Stand-alone Principle: Evaluation of a project based on the project's incremental cash flows.

VERY IMPORTANT: only evaluate **INCREMENTAL** cash flows of project: changes due to accepting

Will this cash flow occur **ONLY** if we accept the project? If yes, use it, if no, **DO NOT**. (If part, use part)

Sunk Cost: a cost that has already been incurred.

DO NOT include in calculations, already happened regardless of project (i.e paid for consulting)

Opportunity costs: The most valuable alternative that is given up if a particular investment is undertaken.

DO include in calculations, if we don't use x for y, what could we get from it?

Change in Net Working Capital (Δ NWC): added current assets – added current liabilities

DO include in calculations, increased NWC means we made investment, subtract from PV (recoup)

We ignore financing decisions for cash flows but include in cost of capital calculations to factor in

“What-If” Project Analysis:

- Sensitivity
 - Effect on NPV when one variable is changed
 - Reveals which variables would drastically change NPV
 - Assumes each variable is independent of each other (not usually true)
- Scenario
 - Good to use when variables are related/interdependent
 - Compares one scenario of variables with another scenario of assumptions.
- Break-even
 - Determines the level of a variable that generates an NPV of zero
 - Often, look at sales volume (Q). If sales higher than break-even → positive NPV.
 - Could also solve for other variables, IRR is really just break-even discount rate

Risk and Return

Steps to find standard deviation

1. Calculate expected return $E(r)$
2. Square expected return $E(r)^2$
3. Calculate $E(r^2)$: sum of [squared return * probability]
4. Variance = $E(r^2) - E(r)^2$
5. Standard Deviation = Square root of Variance ** (IMPORTANT: calculate/use what is asked)

Alternative method: Find $E(r)$, Sum probability(i) [$\{ r(i) - E(r) \}^2$] → variance → square root for std

Covariance: Measures degree which variables move together

$$\text{Covariance} = E(X * Y) - [E(X) * E(Y)]$$

*NOTE: Covariance(x,y) also = correlation(x,y) * variance(x) * variance(y) **see below

Correlation: Normalized value of covariance: [-1 <= correlation <= 1]

$$\text{Correlation} = \text{covariance}(x,y) / [\text{variance}(x) * \text{variance}(y)] \quad \text{**solve for 2nd covariance formula}$$

Recall COMM 291 graphs of what correlation would look like

$f = a * X$	$f = a * X + b * Y$
$E(f) = a * E(X)$	$E(f) = a * E(X) + b * E(Y)$
$\text{Variance } f = a^2 * \text{Variance}(X)$	$\text{Var } f = [a^2 * \text{Var}(X)] + [b^2 * \text{Var}(Y)] + 2 * a * b * \text{correl}(X, Y)$
$\text{Standard Deviation } f = a * \text{std}(X)$	$\text{Standard Deviation } f = \text{square root of Var}(f)$

Risk Averse: All else equal, investors prefer investments with same return and lower variance (risk)

This is known as mean-variance optimization

- Assumes risky assets follow normal distribution
- OR assumes that investors utility is quadratic (with only mean and variance as inputs)

-As such, investors require risk premium to hold risky asset

Since investors like return and hate risk, makes sense to **diversify** away as much risk as possible

In forming a portfolio, diversification benefits stem from correlation between investments

- Generally, the lower the correlation, the greater the diversification benefits
- Be able to read info about assets (return, std, var, covar, correl) and calculate portfolio's return/risk

Lots of risk can be diversified away → Unsystematic risk

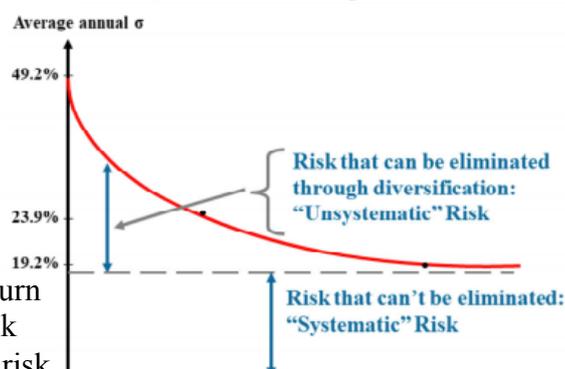
- Unsystematic/diversifiable/unique

Some risk can't be diversified away → Systematic risk

- Systematic/undiversifiable/market

However, investors **will** pick riskier investments, **IF** it comes with the required risk premium.

- So generally, the higher the risk, the higher the return
- BUT**, investors are only “compensated” for systematic risk
- Risk premium is compromised **only** of systematic risk



When constructing portfolio p weights, use w terms which sum to 1
 Calculate $E(p)$ and $\text{std}(p)$ for all different weights $w \rightarrow$ graph on return and risk \rightarrow **Capital Market Line**

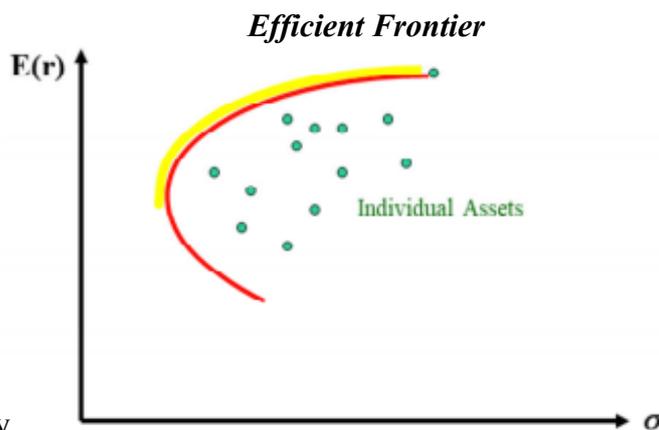
Key findings of doing this...

- Some combination which minimizes risk [**Minimum Variance Portfolio (MVP)**]
- Lots of inefficient combinations with lower return with same risk, or higher risk with same return

Portfolio	$E(r)$	σ
A	9%	12%
B	10%	12%
C	10%	13%
D	14%	24%
E	15%	22%

$B > A, B > C$ $E > D$
 B is efficient E is efficient

Can't say for sure if $B > E$, depends on investors utility



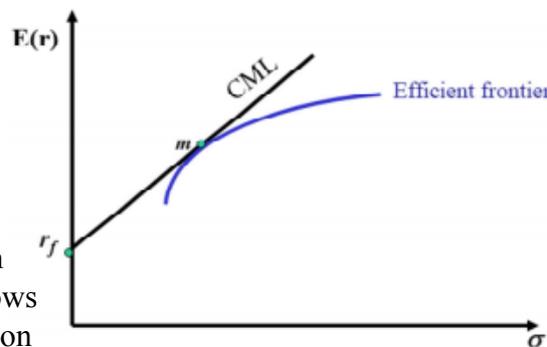
Risk Aversion: How much return an investor requires to take on an additional unit of risk

Now, we introduce the *risk-free* investment (r_f)

- Relatively low return, zero risk (variance)
- Investors can invest, **AND** borrow at this rate

Form the **market portfolio** by combining risk free and portfolio

- Tangent to the efficient frontier at point m
- **Capital Market Line**, investor chooses based on aversion
- To the left of m , investor lends, to the right, investor borrows
- The CML is a straight line because risk-free has no variation



** Why use tangent combinations? This is the highest slope \rightarrow greatest return – risk trade-off

Expected return for market portfolio = $w \cdot E(\text{return of market}) + (1-w) \cdot (\text{return of risk-free})$

- Can be re-written as $E(\text{Return}) = \text{return of risk-free} + w \cdot [E(\text{return on market}) - \text{risk free}]$
 ** This is how we derive CAPM

Variance of market portfolio = $[\text{weight in market portfolio}]^2 \cdot \text{Variance}(\text{market portfolio})$

Standard deviation of market portfolio = $\text{weight in market portfolio} \cdot \text{Standard deviation}(\text{market portfolio})$

Recall earlier graph of efficient frontier and individual assets holdings, since investors only pick market portfolio, the prices of all other individual assets must be priced to be part of the portfolio as well.

- This pricing is determined by the amount of systematic risk the asset contributes
- $E(\text{return on any risky asset}) = \text{risk free} + \beta \cdot [E(\text{return on market}) - \text{risk free}]$ (SML)
 - Where Beta (β) = $\text{Covariance}(i,m) / \text{variance}(\text{market})$
 - Measures stock's systematic risk
 - Tracks how a stock co-moves with the market
 - Creates the Security Market Line (SML), which graphs expected return versus beta

**Note, when $\beta = 1$, SML predicts $E(r_m)$

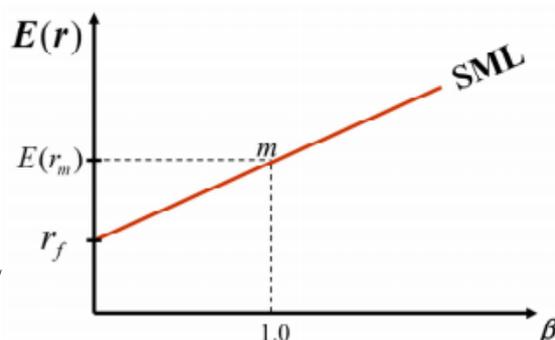
If a stock has high volatility, but low Beta, it isn't "risky"

CML vs. SML

CML graphs excess return / std, explains efficient portfolios

SML graphs expected return / beta, helps price stocks

- "underpriced" is above SML, "overpriced" is below SML
 - Returns to "correct" pricing over time



Beta of a portfolio = weighted average of betas of your stocks

$\beta_{\text{portfolio}} = w_1 \beta_1 + w_2 \beta_2 + \dots + w_n \beta_n$

Cost of Capital

Firms choose financial policy of how to fund their business

- Debt
- Equity

Look at cost of capital through lens of investors:

- **All equity firms**
 - Value of the firm = value of equity
 - Risk of firm = risk of equity
 - Rate of return on firm = return on equity
 - So how much do investors require to give funds? **Return on equity**
 - We already know how to forecast returns of a stock: **SML**
 - Use risk free, market portfolio return, and stock's beta
 - **But what change's a company's beta?**
 - Changes in product line
 - Changes in technology
 - Deregulation
 - Changes in leverage
- **Mix of debt and equity firms**
 - Value of firm = value of equity and debt (Recall COMM 293: $A = L + E$)
 - Rate of return of firm = weighted average of returns on debt and equity
 - So, **Firm's cost of capital = weighted average of debt and equity returns**
 - Call this WACC: Weighted average cost of capital
 - $WACC = (D/V) * R_D + (E/V) * R_E$
 - Find R_E using SML
 - R_D & D/V usually given in debt terms
 - But if old issuance, R_D is really YTM on those bonds
 - Market value of Equity (E) = # of shares * share price
 - Market value of Debt (D) = # of issued bonds * market price
 - $D + E = V$
 - Use to find D/V & E/V for R_{WACC}
 - If firm ALSO issues preferred equity, add $P/V * R_P$ to WACC equation
 - Preferred equity issues fixed constant dividends
 - Effectively a perpetuity!
 - Price today: Next year's Dividend / required return, $P_0 = D_1 / R_P$
 - So, R_P , cost of preferred equity, = D_1 / P_0

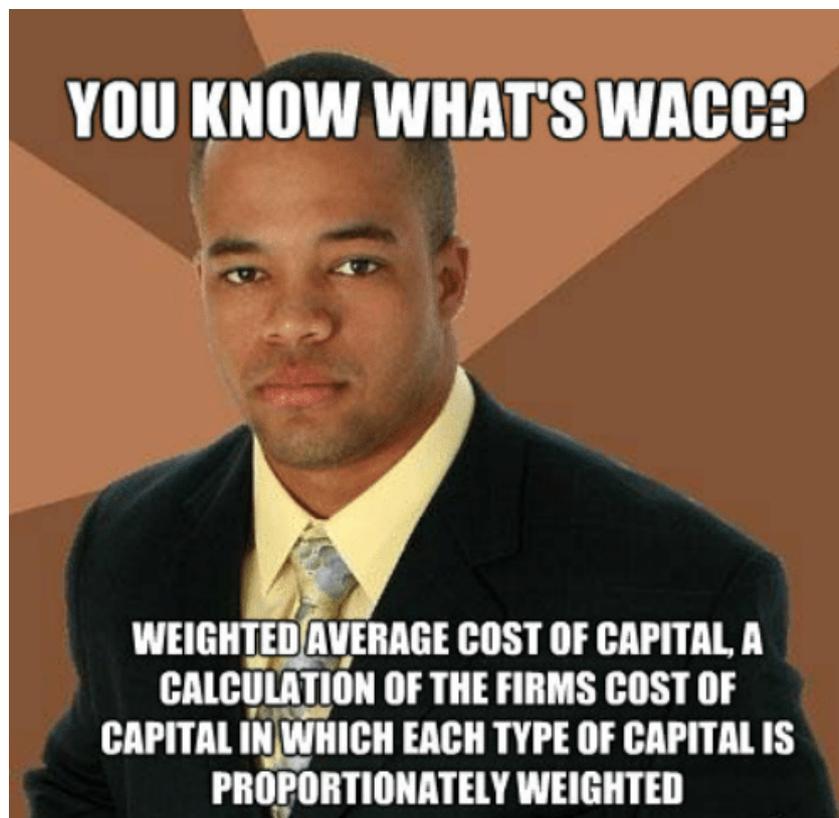
Applying WACC to value project's

- Only when project has similar risk to company's normal operations
- AND, the firm imposes the current capital structure on the project
 - i.e. firm is 50/50 Equity/Debt, fund project costing \$100 by issuing \$50 debt, \$50 of equity
- If project is in unrelated line, or different risk, or impose different structure, DON'T use WACC

5-Minute Break

Next, we'll review the formula sheet and do practice questions

If you have any questions, type them in chat!



COMM 298 Final Exam Formula Sheet – Page 1 of 2

Discounting: $PV = FV(1+r)^{-n}$

Compounding: $FV = PV(1+r)^n$

Present Value of an n-payment annuity of \$A = $A \left[\frac{1 - (1+r)^{-n}}{r} \right]$

Future Value of an n-payment annuity of \$A = $A \left[\frac{(1+r)^n - 1}{r} \right]$

Present Value of a perpetuity of \$A = $\frac{A}{r}$

Coupon in \$ terms: $I = F \times \frac{CR}{k}$

Bond valuation: $P = I \left[\frac{1 - (1+r)^{-n}}{r} \right] + F(1+r)^{-n}$

YTM's relationship to r : $YTM = r \times k$

Realized Rate of Return: $ROR = \left[\frac{(\text{selling price} + FVRC)}{\text{purchase price}} \right]^{\frac{1}{n}} - 1$

Constant Dividend Growth: $P_0 = \frac{DIV_1}{r-g}$ and so $r = \frac{DIV_1}{P_0} + g$

Change in NWC = $\Delta NWC = \Delta \text{current assets} - \Delta \text{current liabilities}$

Profitability Index = $\frac{PV \text{ of Future Cashflows}}{\text{Initial Investment}}$

COMM 298 Formula Sheet – page 2 of 2

Expected Value: $E(X) = \sum x \cdot p(x)$

Variance: $VAR(X) = \sigma_X^2 = E[X^2] - [E(X)]^2$

Covariance: $\sigma_{XY} = E(X \cdot Y) - [E(X) \cdot E(Y)]$

Correlation: $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$ and so $\sigma_{XY} = \rho_{XY} \sigma_X \sigma_Y$

$$E(aX) = aE(X)$$

$$VAR(aX) = a^2 \sigma_X^2$$

$$E(aX + bY + cZ) = aE(X) + bE(Y) + cE(Z)$$

$$VAR(aX + bY + cZ) = a^2 \sigma_X^2 + b^2 \sigma_Y^2 + c^2 \sigma_Z^2 + 2ab \sigma_{XY} + 2ac \sigma_{XZ} + 2bc \sigma_{YZ}$$

Portfolio on CML has: $\sigma_p = w_m \sigma_m$

SML: $E(r_i) = r_f + \beta_i [E(r_m) - r_f]$

Beta: $\beta_i = \frac{\text{Cov}(r_i, r_m)}{\sigma_m^2}$

Portfolio Beta: $\beta_P = w_1 \beta_1 + w_2 \beta_2 + \dots + w_N \beta_N$

Weighted Average Cost of Capital: $WACC = \left(\frac{D}{V}\right) \times R_D + \left(\frac{P}{V}\right) \times R_P + \left(\frac{E}{V}\right) \times R_E$

Sample Questions

Question 1.

Do these statements provide evidence **against** the **semi-strong form** Efficient Market Hypothesis?

- a) Every time Tesla announces their quarterly earnings report, investors under-react to the results
*Prices should factor in all public information, this is evidence **against** semi-strong form EMH.*
- b) Elon Musk privately tells his employees Tesla is going bankrupt, and the share price is unaffected
*This is private information, so it should not affect the share price → evidence **for** semi-strong form*

Question 2.

For each project scenario, what would be the **best** investment decision rule to use?

- a) Ford has to choose between 2 mutually exclusive, 5-year projects to produce limited-time cars
NPV, we need to see value added to the firm, and IRR would be distorted by the cross-over rate
- b) Apple has a set required rate of return for all of their projects of 10%
IRR, if we calculate the project's IRR we can simply compare it to their target to decide
- c) A lemonade stand owner only has \$50 to invest in equipment. They calculate the future benefits and current costs of getting another juicer, or an extra sign, or hiring a friend to sell on other streets
*PI, since capital is scarce, the owner wants the most relatively profitable investment per \$1 cost
Could also argue **payback period** since the stand owner likely doesn't care about TVM*

Question 3.

Assume the following information about stocks A and B:

$E(R_A) = 10\%$, $(\sigma_A) = 5\%$, $E(R_B) = 8\%$, $(\sigma_B) = 6\%$

- a) If you could only invest in one stock, which one would you choose?
Stock A, it offers higher expected return AND lower risk than Stock B
- b) If you had a portfolio of many stocks with $E(R_P) = 9\%$ $\sigma_P = 6\%$ would you invest in A or B?
*Stock A offers higher return and lower risk, so it **would be added**. Depending on the correlation of stock B to the rest of your portfolio, Stock B may diversify risk and be worth investing in.*
- c) Can you calculate expected return and variance of a portfolio with just 75% in A, and 25% in B?
 $E(R_P) = [(3/4) * 10\%] + [(1/4) * 8\%] = 9.5\%$
 (σ^2_P) = is **unknown** because you are not given covariance or correlation OR probabilities of returns

Question 4.

Imagine the expected market return is 8%, with standard deviation of 3%, and the risk-free return is 2%.

- a) What would the return of stock Q with a β of 1.2 be?

$$\text{Use CAPM: } E(R_Q) = .02 + 1.2 * (.08 - .02) \rightarrow E(R_Q) = 9.2\%$$

- b) If you wanted an expected return of 6%, what would the optimal weights in your portfolio be?

$$\begin{aligned} 0.06 &= [W_M * R_M] + [(1-W_M) * R_{r.f}] \\ 0.06 &= 0.08 * W_M + .02 * (1-W_M) \\ 0.06 &= .08 * W_M + .02 - .02 * W_M \\ W_M &= 66.67\% \end{aligned}$$

- c) What would be the variance of your portfolio in part b) ?

$$\begin{aligned} \sigma_P^2 &= W_M^2 * \sigma_M^2 \\ \sigma_P^2 &= (.6667)^2 * (.03)^2 \\ \sigma_P^2 &= 0.0004 \text{ or } 0.04\% \end{aligned}$$

Question 5.

Suppose you the following portfolio, the expected market return is 7.5%, and the risk-free return is 1%

<u>Stock</u>	<u>Value</u>	<u>Beta</u>
Tesla	\$1000	1.3
Apple	\$700	0.8
Risk-free	\$300	0.0

- a) What is the expected return of your portfolio?

$$\begin{aligned} \text{First find portfolio } \beta &\rightarrow [(1000/2000) * 1.3] + [(700/2000) * 0.8] + [(300/2000) * 0] = 0.93 \\ \text{Use CAPM: } E(R_P) &= .01 + 0.93 * (.075 - .01) \rightarrow E(R_P) = 7.045\% \end{aligned}$$

- b) If Tesla's risk was under-stated, and actually had a market beta of 2.0, how does $E(R_P)$ change?

*Since the Beta of the portfolio increases, we would expect the portfolio return to **increase***
***NOTE: since we are using Beta, we assume the SYSTEMATIC risk was under-stated*

Question 6.

Stock ABC is currently trading at \$50. The dividend which was just paid was \$2.50. The dividend growth rate this year will be 5%, next year it will be 1%, and then it will settle forever at a growth rate of 2%.

- a) Assuming ABC's investors required rate of return is 8%, is this stock correctly priced?

$$\begin{aligned} PV(D_1) &= (2.50 * 1.05) / 1.08 \rightarrow \$2.43 \\ PV(D_2) &= (2.50 * 1.05 * 1.01) / (1.08)^2 \rightarrow \$2.273 \\ \text{Terminal Value as of year 2} &= (2.50 * 1.05 * 1.01 * 1.02) / (.08 - .02) = \$45.07 \\ PV(TV_2) &= 45.07 / (1.08)^2 \rightarrow 38.641 \\ \text{Sum of PV's} &= 2.43 + 2.274 + 38.641 \rightarrow \text{Price today should be } \$43.34, \text{ so over-priced.} \end{aligned}$$

5-Minute Break

Please fill out the survey on the FB page!

Here's a challenge question to work on...

You took out a small payday loan from Santa Ono and forgot about it for 2 years. Since the EAR was so high, you now owe him \$4500! Santa feels bad, so he will *only* charge a 33.33% EAR this year.

You look at the market, and see that oil prices are extremely volatile, especially in a high-risk mining company where $E(R) = 20\%$, and $\sigma = 40\%$. Trying to help, your mom gives you \$3000.

- a) Assuming you **must** pay back Santa Ono **one year from today**, would it be better to invest all \$3000 into the oil company, or invest only \$1500 and pay back \$1500 to Santa Ono today?

*If you invest all \$3000, you will owe Santa Ono $(4500 * 1.3333) = \$6,000$ at the end of the year.
To end the year with \$6,000 you would need a return of $(6000/3000) - 1 = 100\%$
Since returns are normally distributed, the chances of a 100% return are $\sim .15\%$ (68-95-99.7)*

*If instead, you pay down \$1500 today, you will owe Santa $(3000 * 1.3333) = \$4,000$ in one year.
To end with \$4000 on your \$1500 investment, you need a return of $(4000/1500) - 1 = 166.67\%$
Since returns are normally distributed, the chances of a 166.67% return are even lower.*

Therefore, you should pick to invest all \$3,000 instead of paying down \$1,500 today.



More Sample Questions

Question 7.

The Subway on-campus is planning a 10-year project so students don't have to wait in line for 30 minutes. They paid some COMM 204 students \$5,000 to design a new mobile queuing system, and found some promising results. If they take on the project, it will have the following impact:

- Initial investment in software required of \$800,000
- Three additional payment machines required, which would cost \$7,500 each
- Triple the usual annual Net Operating Cash Flows of \$70,000

At the end of the project, Subway plans to sell the software to a tech start-up for \$100,000. Also, they will sell the 3 new payment machines back to a wholesaler, but the machines will only be worth half as much. Subway uses a discount rate of 5% to value this project, and does not pay taxes.

- a) What is the present value of the salvage value of the payment machines?

$$\begin{aligned} \text{Value of payment machines at end of project} &= 3 * (.5 * 7500) = \$11,250 \\ \text{PV of salvage from machines} &\rightarrow 11,250 / (1.05)^{10} = \mathbf{\$6,906.52} \end{aligned}$$

- b) What is the NPV of this project?

$$\begin{aligned} \$5,000 \text{ design cost is a sunk cost, ignore in calculations.} \\ \$800,000 + 3 * \$7,500 &= \text{Initial investment of } (\$822,500) \\ \text{Incremental change in Annual Net Operating Cash Flow} &= (3 * 70,000) - 70,000 = +\$140,000 \\ \text{Salvage at end of project} &\rightarrow 100,000 + [3 * (7,500)] / 2 \rightarrow +\$111,250 \end{aligned}$$

$$\begin{aligned} \text{PV of 10-year annuity of Net Operating CF} &\rightarrow 140,000 * [(1 - (1.05)^{-10}) / .05] = \$1,081,042.89 \\ \text{PV of salvage at end of project} &\rightarrow (100,000 + 11,250) / (1.05)^{10} = \$68,297.85 \\ \text{PV of all benefits} &\rightarrow 1,081,042.89 + 68,297.85 = \$1,149,340.74 \end{aligned}$$

$$\text{NPV} = \text{PV Benefits} - \text{Initial Costs} \rightarrow 1,149,340.74 - 822,500 = \mathbf{NPV = \$326,840.74}$$

- b) What is the Profitability Index of this project?

$$\text{PI} = \text{PV of benefits} / \text{initial costs} \rightarrow 1,149,340.74 / 822,500 = \mathbf{1.397}$$

- c) Currently, we expect this project to triple annual Net Operating Cash Flows. For this project just to break-even, by how much would annual Net Operating Cash Flows have to increase?

$$\begin{aligned} \text{For breakeven, NPV} &= 0 \rightarrow \text{PV of benefits must be = to Initial Investment} \\ \text{PV of annuity of additional Net Operating Cash flows} &+ \text{PV of both salvages} = \text{Initial Investment} \\ \text{PV of annuity} &+ 68,297.85 = 822,500 \\ \text{PV of annuity must} &= \$754,202.15 \\ 754,202.15 &= x * [(1 - (1.05)^{-10}) / .05] \\ 754,202.15 &= x * [7.72173] \\ x &= \$97,672.63 \end{aligned}$$

To break even, annual Net Operating Cash Flows must increase by \$97,672.63 or by 139.53%

Question 8.

Joe's Bagel Company is looking to take on a project to start selling donuts. Joe paid a consultant \$10,000 to forecast the selling price and cost of making donuts, and expects to incur \$2 of variable cost per unit, and sell them at his store for \$6 each. Thankfully, Joe inherited his equipment, so he has no fixed costs. If his company takes on this 5-year project, it will have the following impact:

- Investment in equipment required today of \$90,000, depreciated straight-line, no salvage in 5 years
- Investment needed in extra materials today of \$15,000 to be recovered at the end of the project.

Joe's Bagel Company currently has 75,000 shares outstanding, with a share price of \$8. Additionally, they have 500 bonds outstanding, at \$1,000 face value each, with 15 years to maturity, and a YTM = 6.5%. The total market value of these bonds is equal to \$400,000. Their stock's Beta = 0.6, the market return is 9%, and the risk-free rate is 3%. Joe's Bagel Company does not pay taxes.

- a) Assuming the project has normal operations risk, what is the appropriate discount rate?

Mixture of Debt and Equity → Discount by WACC

$E(R_{Joe}) = .03 + 0.6 * (.09 - .03)$ Required rate of return on equity = 6.6%

YTM on debt = 6.5% → Required rate of return on debt = 6.5%

Value of Equity = 75,000 shares * \$8 share price = \$600,000

Value of Debt = \$400,000

$WACC = [(600,000 / 1,000,000) * 0.066] + [(400,000 / 1,000,000) * 0.065] = 6.56\%$

- b) What is the **total** annual coupon Joe's Bagel Company issues on the company's bonds?

$\$400,000 = I * [(1 - (1 + .065)^{-15}) / .065] + [(500 * 1,000) / (1 + .065)^{15}]$

$\$400,000 = (I * 9.40267) + 194,413.2622$

$I = \text{Annual Coupon} = \$21,864.72$

- c) Joe is uncertain of donut demand due to COVID-19. What is **the project's** NPV breakeven, in terms of annual quantity of donuts sold?

Need to eventually find annuity value so that PV of benefits = Initial Costs

PV of 5-year incremental annuity + PV of NWC recovery = Initial Cost

$PV \text{ of annuity} + [15,000 / (1.0656)^5] = \$105,000$

PV of annuity must be equal to \$94,082.575

$PV \text{ of annuity} = c * [(1 - (1 + .0656)^{-5}) / .0656]$

$PV \text{ of annuity} = c * 4.1489$

$c = \$22,676.19 \text{ needed}$

*Operating cash flow from donut sales = $(q * 6) - (q * 2) \rightarrow 4q \rightarrow$ they make \$4 per donut sold*

*To make \$22,676.19 extra a year, they would need to sell $22,676.19 / 4 = 5,669.05$ or **5,669 donuts***

Q&A

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