commerce undergraduate society

MATH 104/184

MIDTERM REVIEW SESSION

by Peter Im

Concepts

Limits

The limit of a function is a value that the function **approaches** as the input **approaches** some value.

$$\lim_{x \to a} f(x) = c$$

This reads, "The limit of f(x) as x approaches a is equal to c"

We say that the limit exists as x -> a when our function approaches the same value from both sides, that is, when:

$$\lim_{x \to a+} f(x) = \lim_{x \to a-} f(x)$$

When evaluating a limit, your first step should be to plug in the value of x = a into your original function. If this works and gives you a numerical answer, no further work is required. Sometimes, you may end up plugging in x = a into a fraction that gives you the value of 0/0. This is called an **indeterminate form**. To get information from one of these indeterminate forms, some algebraic manipulation is usually required. (Covered in examples!)

Continuity

A function f is continuous at x = a when:

1) f(a) is defined
2) $\lim_{x \to a} f(x)$ exists
3) $\lim_{x \to a} f(x)$ = f(a)

In other words, the graph of the function is unbroken i.e. you could draw it from any point to another without lifting a pen from your paper.

It's important to note that many types of functions, such as polynomial functions, sin(x), cos(x) are continuous throughout their entire domains. Since proving the continuity of a function for a given domain is outside the scope of this course, you can state that these functions are continuous without providing proofs.

Intermediate Value Theorem

Theorem: If f is a continuous function whose domain is in the interval [a,b], then there exists at least one c within [a,b] such that f(a) < f(c) < f(b).

Derivatives

The derivative is the **instantaneous rate of change** of a function at a specific point.

The derivative of f(x) is given by

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Provided that the limit exists. We say that f(x) is **differentiable** where this limit exists.

f'(a), the derivative of f(x) at x = a can also be described by

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}.$$

A useful property that the derivative has is that its value at x = a is equal to the slope of the line that is tangent to f(x) at x = a.

In other words,

$f'(a)$ = the slope of the line tangent to f(x) at x = a

1. Short Problems. Put your answer in the box provided and show your work. No credit will be given for an answer without the accompanying work.

(a) Evaluate $\lim_{x\to 7}\left(\frac{x^2-4x-21}{3x^2-17x-28}\right)$

Answer:

(b) Evaluate $\lim_{x\to -3}\left(\frac{\sqrt{2x+22}-4}{x+3}\right)$

Answer:

(c) Evaluate $\lim_{x\to 1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right)$

Answer:

(d) Let $h(x) = e^{3f(x)} + [f(x)]^2$, $f(1) = 2$, $f'(1) = 5$

Find $h'(1)$

Answer:

Long Problems. No credit will be given for the answer without the correct accompanying work.

2. Find the equations of the lines parallel to $2y - 2x = 2019^{2020}$ and tangent to the graph of $y = 2x + 3x^{-1}$

3.

(a) Carefully state the limit definition of the derivative of the function $y = f(x)$

(b) Use the limit definition of the derivative to find $f'(9)$ for the following function. No marks will be given for the use of differentiation rules.

$$f(x) = -6\sqrt{x}$$

4. Find the values of a and b such that $f(x)$ is continuous for all real numbers. With these values, is $f(x)$ differentiable at x = 1?

$$f(x)\begin{cases} e^{x}+a & \text{if} \quad x>2 \\ bx^{2}+1 & \text{if} \quad 1\leq x\leq 2 \\ 3x^{3}-b & \text{if} \quad x<1 \end{cases}$$

5. Professor Fenceman, an economics professor at the University of Building Connections sells 200 of his award-winning review packages at $110 each. He rents an industrial-grade printer at $50 per term and each review pack costs $70 to produce. Dr. Fenceman's students are price-sensitive, and some will decide not to buy the review package if the price is too high. He estimates that for every $20 increase in his review package price, 20 students will choose not to purchase it.

(a) Find the linear demand function for the review package as a function of q (quantity).

(b) Find the Revenue function as a function of q (quantity)

(c) What should Dr. Fenceman charge to maximize profit?

6. Prove that the equation below has a root.

$$f(x) = 4x^4 - 16x^3 + 2x^2 - x + 9$$

7. Find the equation of the line that passes through the origin and is tangent to the graph of $f(x) = \ln x$

Appendix: Formulae

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x).$$

$$\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x).$$

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x).$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \quad \text{if} \quad \lim_{x \to a} g(x) \neq 0.$$

$$\lim_{x \to a} b^{f(x)} = b^{\lim_{x \to a} f(x)},$$

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$